Collectively Simplifying Trajectories in a Database: A Query Accuracy Driven Approach (Technical Report)

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***Abstract*—Increasing and massive volumes of trajectory data are being accumulated that may serve a variety of applications, such as mining popular routes or identifying ridesharing candidates. As storing and querying massive trajectory data is costly, trajectory simplification techniques have been introduced that intuitively aim to reduce the sizes of trajectories, thus reducing storage and speeding up querying, while preserving as much information as possible. Existing techniques rely mainly on hand-crafted error measures when deciding which point to drop when simplifying a trajectory. While the hope may be that such simplification affects the subsequent usability of the data only minimally, the usability of the simplified data remains largely unexplored. Instead of using error measures that indirectly may to some extent yield simplified trajectories with high usability, we adopt a direct approach to simplification and present the first study of query accuracy driven trajectory simplification, where the direct objective is to achieve a simplified trajectory database that preserves the query accuracy of the original database as much as possible. Specifically, we propose a multi-agent reinforcement learning based solution with two agents working cooperatively to collectively simplify trajectories in a database while optimizing query usability. Extensive experiments on four real-world trajectory datasets show that the solution is capable of consistently outperforming baseline solutions over various types of queries with different dynamics.**

***Index Terms*—trajectory data, trajectory simplification, query processing, reinforcement learning**

I. INTRODUCTION

A trajectory is a sequence of time-stamped locations that describe the movement of an object over time. Massive amounts of trajectory data are being accumulated and used in diverse applications, such as discovering popular routes in a city [1], analyzing animal migration patterns [2], and performing sports analytics [3]. The accumulation of trajectory data [4] introduces at least two challenges [5], [6]: (1) storing the data is expensive, and (2) querying the data is time-consuming. These challenges can be addressed by conducting trajectory simplification, which aims to drop points from trajectories to save the storage cost and speed up query processing. The underlying rationale is that not all points in a trajectory carry equally important information, so that dropping unimportant ones may be acceptable. For example, if the location of an object is sampled regularly and the object does not move for a while then only the first and last positions during the period of inactivity are important, and those in-between may be dropped without loosing information. Next, the efficiency of query processing is improved, at the expense of query results becoming approximate.

Indeed, the extent to which a collection of simplified trajectories enables accurate query results has been used widely as a measure of the quality of a simplification technique in empirical studies of trajectory simplification [7], [5], [6]. For example, Zhang et al. [6] evaluate existing simplification techniques in terms of their ability to produce simplified trajectories that affect the accuracy of range, kNN, and join queries as well as clustering minimally. While there are many proposals for trajectory simplification [8], [9], [10], [11], [12], [13], [14], they all assume a storage budget and aim to produce simplified trajectories that minimize the difference from the original trajectories according to a given difference notion. No proposals aim to optimize directly the query accuracy offered by the simplified trajectories. We call this line of study *Error-Driven Trajectory Simplification (EDTS)*.

In addition, existing simplification techniques are local in nature and operate on a per-trajectory basis as opposed to being global in nature and operating on a database of trajectories. Specifically, they aim to simplify a given trajectory T within a budget $r \cdot |T|$, where $r \in (0, 1]$ is the compression ratio. When these techniques are used to simplify a database of trajectories according to a compression ratio r, they simplify each trajectory in the database *separately* according to compression ratio r. This is likely sub-optimal in cases where trajectories have different sampling rates or different complexities. Intuitively, trajectories with higher sampling rates or lower complexity are candidates for simplification with larger compression ratios.

Therefore, we propose a new trajectory simplification problem, called *Query accuracy Driven Trajectory Simplification (QDTS)*. Given a trajectory database D and a storage budget, the problem is to find a simplified trajectory database D' that preserves the accuracy of query results as much as possible, compared to the query results on D. QDTS differs from the existing EDTS problem. First, it considers a different objective of trajectory simplification, namely that of preserving query accuracy directly, rather than through minimizing an error measure, as in the EDTS problem. Second, it is global in

nature and takes a trajectory database as input and outputs a simplified trajectory database that satisfies a specified storage budget as a whole, instead of simplifying each trajectory in isolation according to a budget, as do existing EDTS techniques [8], [9], [4].

Challenges. An immediate solution to the QDTS problem is to reduce it (i.e., a *database-level* simplification problem) to a *trajectory-level* problem by applying simplification to each trajectory *separately* with a proportional budget. Specifically, it simplifies each trajectory T with the budget of $r \cdot |T|$. It is obvious that the resulting simplified database would contain M simplified trajectories and have at most $r \cdot \sum_{T \in D} |T| = r \cdot N$ points, where N denotes the total number points in the database. While this solution needs minimal design efforts, it suffers from two main issues. Issue 1: Uniform compression ratio: It applies the same compression ratio to each trajectory, which would be sub-optimal for trajectories with different sampling rates and/or different complexities. Issue 2: Query accuracy unawareness: Existing algorithms [8], [9], [10], [11], [12], [13], [14] (all of which operate at trajectory-level) aim to optimize some form of error metric that quantifies the difference between an original and a simplified trajectory. As a consequence, simply using any of these algorithms cannot help to optimize directly the query accuracy of the simplified database.

Another solution is to consider all trajectories in the database D *collectively* during the course of simplification. Consider a top-down approach, in which we start with the most simplified database, i.e., each simplified trajectory T' of an original trajectory T contains only the first and last points of T. We then iteratively introduce points from the original database according to some selection criterion until the budget is exhausted. Alternatively, we can adopt a bottom-up approach, in which we start from D and iteratively drop points until the remaining points are within the budget. This solution considers all trajectories *collectively* when simplifying trajectories, enabling different trajectories to be simplified with different compression ratios depending on their complexities. Therefore, it avoids the first issue mentioned above. However, this solution still does not contend with the second issue. Furthermore, the solution operates at the database level, whose scale is typically much larger than that of a single trajectory. For example, the Geolife dataset used in our experiment contains millions of points, while individual trajectories contain only around one thousand points. This brings up a third issue. Issue 3: Lack of scalability: Operating at the database level, the solution needs to repeatedly choose a point from among a very large set of points.

New Solution. Motivated by the above discussion, we propose a new solution called `RL4QDTS` for trajectory database simplification, which avoids all the three issues. `RL4QDTS` has two core ideas. First, it considers all trajectories in the database *collectively* for simplification. Specifically, it starts with the most simplified database, then introduces original points into the simplified database iteratively until its budget is exhausted. For better efficiency, it uses an index that partitions the database into sub-spaces, called *spatio-temporal cubes*. Whenever it needs to choose a point to introduce into the database, it first chooses a cube based on the index and then chooses a point in the cube. To partition a database of trajectories, one immediate idea is to partition along the spatial and temporal dimensions with a predefined granularity, e.g., setting a grid size for the spatial dimensions and a time duration for the temporal dimension. Nevertheless, the granularity is hard to set appropriately and is unlikely to work across databases. Small cubes (corresponding to a fine granularity) contain few candidate points, making it difficult to find good points to introduce. Large cubes contain many points, making it costly to choose one point within a cube. Therefore, `RL4QDTS` builds an octree (a three-dimensional variant of the quadtree for spatio-temporal points) to partition the database into cubes. The octree provides different resolutions of data cubes organized in a tree structure, making it is possible to choose cubes with different sizes flexibly and adaptively by traversing the tree from the root node to an appropriate node. We adopt the octree for its simplicity and leave other indexes, e.g., kd-tree [15], for future exploration.

Second, `RL4QDTS` leverages multi-agent reinforcement learning to choose a point iteratively such that the query accuracy based on the simplified database involving the chosen points is optimized. Specifically, it employs an agent (called Agent-Cube) to find an octree node with a cube of an appropriate size. Then, it employs another agent (called Agent-Point) to choose a point in the cube chosen by Agent-Cube and introduces the point into the simplified database. Specifically, the two agents employ Markov decision processes (MDP) [16] that are designed such that the two agents optimize cooperatively the query accuracy on the simplified database. The `RL4QDTS` algorithm then leverages the learned polices of the two agents for simplifying a trajectory database. In summary, the first idea enables a solution that avoids the first and third issues, and the second idea is to address the second issue.

Overall, we make the following contributions.

- We propose the QDTS problem that aims to find a simplified trajectory database within a given storage budget that preserves the query accuracy on the simplified database as much as possible. This is the first systematic study of this line of trajectory simplification. (Section III)
- We develop a multi-agent reinforcement learning based solution called `RL4QDTS` to the problem. It simplifies a database of trajectories collectively with the aim of optimizing the query accuracy on the simplified database, while leveraging an index on the trajectory data for better efficiency. We show that the objective of `RL4QDTS` is well aligned with that of the QDTS problem. (Section IV)
- We conduct experiments on four real-world trajectory datasets, showing that `RL4QDTS` consistently outperforms existing EDTS solutions across *two* types of adaptions, *four* error measures, and *varying* storage budgets for *five* query operators. For example, it achieves the improvement of up to 35% for range query, 41% and 28%

for two kinds of kNN query, 35% for similarity query and 40% for clustering than the best baselines. (Section V)

II. RELATED WORK

Error-Driven Trajectory Simplification. The error-driven trajectory simplification aims to simplify a trajectory within a given storage budget and to minimize an error measure of the simplified trajectory. Many studies have been conducted on this problem, among which some focus on the batch mode (where full access to a trajectory is attained throughout the process) [8], [9], [10], [11] and others on the online mode (where a trajectory is inputted in an online fashion and those points that have been dropped are no longer accessible) [12], [13], [14]. We review the studies on the batch mode, which is the focus of this paper, as follows. Specifically, Top-Down [8] adapts the traditional Douglas-Peucker algorithm [17]. The algorithm starts with two points (the first and last) of a trajectory. Then, it repeatedly inserts a point with the largest error until the size of the simplified trajectory reaches the storage budget. Bottom-Up [9] adopts a reverse strategy. It scans all points of the input trajectory and repeatedly drops the point with the smallest error until the number of remaining points is within the storage budget. Long et al. [10] propose Span-Search, which is designed specifically to preserve direction information in trajectory simplification. Recently, Wang et al. [11] propose a reinforcement learning based method called RLTS+ for trajectory simplification. It adopts the Bottom-Up strategy and drops points based on a learned policy instead of using the heuristic rules seen in previous studies.

Overall, the above studies aim to minimize a given error measure while simplifying a trajectory. However, they largely disregard data usability as an objective of simplification algorithms. As a matter of fact, one of the main motivations for simplification is to improve query efficiency. Consequently, data usability should be treated as a key factor to indicate the quality of trajectory simplification. Indeed, data usability has been used to compare existing simplification algorithms in several empirical studies [7], [6], [5]. An early study [7] evaluates several error measures in trajectory simplification and analyzes the soundness of the measures for queries. Zhang et al. [6] consider four spatio-temporal queries (i.e., range query, kNN query, join query, and clustering) on a trajectory database and design the corresponding measures to evaluate the quality of existing trajectory simplification algorithms. A recent evaluation study [5] verifies the query qualities of error-bounded trajectory simplification algorithms [18], [19], [20], [21] that simplify a trajectory with a given error tolerance and aim to minimize the size of a simplified trajectory. However, data usability is only used as evaluation measures in these studies [7], [6], [5] to understand how well the existing simplification algorithms support various types of queries, but not considered in simplification algorithm design. We note that existing optimal algorithms [22] for EDTS problem cannot be applied or adapted to the QDTS problem studied in this work. They are usually dynamic programming based or binary search based and have high time costs (i.e., cubic time complexity for quite a few error measures), and thus they are not practical.

Other Types of Trajectory Simplification. Other studies of trajectory simplification include: (1) studies simplifying a trajectory such that the error of the simplified trajectory is bounded and as many points as possible are dropped, considering batch mode [23], [8], [9], [14] and online mode [4], [18], [23], [20], [21], (2) studies simplifying a trajectory by immediately deciding whether to keep or drop an incoming point (also called *dead reckoning*) [12], [13], [14], (3) a study which develops a trajectory quantization method that assigns a smaller number of bits for each trajectory point - when it assigns 0 bits to a point, it means to drop the point [24], and (4) a study which develops optimal algorithms for the curve simplification under different settings of distance/error measures and restrictions of points to be kept [25]. These studies do not return trajectories with sizes bounded by user-specified parameters and thus cannot be used for our QDTS problem.

Road Network-based Trajectory Compression. Road network-based trajectory compression [26], [27], [28], [29] aims to compress trajectories that are generated by objects in road networks. Specifically, raw trajectories are initially map-matched to an underlying road network to obtain map-matched trajectories that consist of sequences of road segments. The map-matched trajectories are treated as strings, where each road segment is considered as a character. Then, string compression algorithms such as Huffman coding [30] can be utilized to compress the trajectories with or without information loss. In contrast, we aim to reduce trajectory data in its original form, i.e., as a sequence of time-stamped locations, without the input of a road network.

Reinforcement Learning. Reinforcement Learning (RL) aims to guide agents on how to take actions to maximize a cumulative reward in an environment, where the environment is usually modeled as a Markov decision process (MDP), involving states, actions, and rewards [16]. Recently, RL has been applied successfully to solve many algorithmic problems, such as similarity search [31], index learning [32], fleet management [33], and trajectory simplification [11], [4]. Our study differs from the existing studies of RL-based trajectory simplification in three aspects. 1) Our method simplifies trajectories collectively in a database, rather than simplifying each trajectory with an uniform compression ratio [11]. 2) Our method optimizes the query accuracy on a simplified database, rather than minimizing an error measure on a single trajectory [11] or minimizing the size of a simplified trajectory [4]. Both existing methods are query un-aware. 3) Our method builds an octree on the trajectory database and iteratively chooses a point by choosing *a cube* with a traversal on an octree and then choosing *a point* within the cube. It leverages two agents for the two decision making processes of choosing a cube and a point. These decision making processes are different from those in the existing methods [11], [4] and the corresponding designs (e.g., those of MDPs) are different.

III. PRELIMINARIES AND PROBLEM STATEMENT

A. Preliminaries

Trajectories and Segments. A *trajectory* T is a sequence of time-stamped points: $T = \langle p_1, p_2, ..., p_n \rangle$, where n is the length of T (i.e., $n = |T|$). Each point p_i $(1 \leq i \leq n)$ is a triple $p_i = (x_i, y_i, t_i)$, indicating that the moving object is at location (x_i, y_i) at time t_i. We define the line $\overline{p_i p_{i+1}}$ linking two neighboring points as a segment in the trajectory. Thus, the trajectory T corresponds to a sequence of segments $\overline{p_1 p_2}, \overline{p_2 p_3}, ..., \overline{p_{n-1} p_n}$. A *trajectory database* D consists of a set of trajectories. We define N to be the total number of points in D.

Trajectory Simplification and Errors. *Trajectory simplification* aims to eliminate points from a trajectory T to obtain a simplified trajectory T' of the form $T' = \langle p_{s_1}, p_{s_2}, ..., p_{s_m} \rangle$, where $m \leq n$ and $1 = s_1 < s_2 ... < s_m = n$. We similarly call the line $\overline{p_{s_i} p_{s_{i+1}}}$ linking two neighbouring points in the simplified trajectory as a simplified segment. The simplified trajectory T' indicates that the object moves along a simplified segment $\overline{p_{s_j} p_{s_{j+1}}}$ $(1 \leq j \leq m-1)$, which approximates the movement along a sequence of segments $\overline{p_{s_j} p_{s_j+1}}, \overline{p_{s_j+1} p_{s_j+2}}, ..., \overline{p_{s_{j+1}-1} p_{s_{j+1}}}$ as indicated by the original trajectory T. Thus, we call the simplified segment $\overline{p_{s_j} p_{s_{j+1}}}$ an *anchor segment* for each of the points $p_{s_j}, p_{s_j+1}, .., p_{s_{j+1}-1}$.

To measure the information loss of the simplification, several *error measures* have been proposed, including Synchronized Euclidean Distance (SED) [12], [13], [14], [21], Perpendicular Euclidean Distance (PED) [21], [20], [34], [35], Direction-aware Distance (DAD) [36], [37], [10], [23], and Speed-aware Distance (SAD) [14]. These measures are defined in two steps. First, the error of a simplified segment $\overline{p_{s_j} p_{s_{j+1}}}$ (denoted by $\epsilon(\overline{p_{s_j} p_{s_{j+1}}})$) is defined as the maximum error of an original point p_i that takes the segment as its anchor segment (denoted by $\epsilon(\overline{p_{s_j} p_{s_{j+1}}}|p_i)$):

$$\epsilon(\overline{p_{s_j} p_{s_{j+1}}}) = \max_{s_j \leq i < s_{j+1}} \epsilon(\overline{p_{s_j} p_{s_{j+1}}}|p_i), \tag{1}$$

where $\epsilon(\overline{p_{s_j} p_{s_{j+1}}}|p_i)$ can be instantiated with SED, PED, DAD, or SAD. Figure 1 illustrates $\epsilon_{SED}(\overline{p_1 p_3}|p_2)$, $\epsilon_{PED}(\overline{p_1 p_3}|p_2)$, and $\epsilon_{DAD}(\overline{p_1 p_3}|p_2)$. Detailed definitions can be found in an evaluation paper [6]. Second, the error of the simplified trajectory T' (denoted by $\epsilon(T')$) is defined as the maximum error of its simplified segments:

$$\epsilon(T') = \max_{1 \leq j \leq m-1} \epsilon(\overline{p_{s_j} p_{s_{j+1}}}) \tag{2}$$

Error-Driven Trajectory Simplification (EDTS). In the EDTS problem [8], [9], [11], [10], given a trajectory $T = \langle p_1, p_2, ..., p_n \rangle$ and a storage budget W, it aims to find a simplified trajectory $T' = \langle p_{s_1}, p_{s_2}, ..., p_{s_m} \rangle$ such that $|T'| \leq W$ and $\epsilon(T')$ is minimized, where $\epsilon(T')$ is SED, PED, DAD, or SAD.

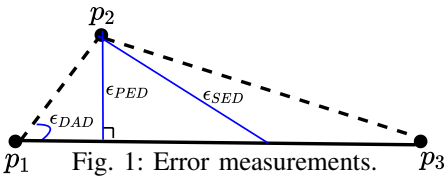


Fig. 1: Error measurements.

B. Problem Definition

We define a new problem, called Query Accuracy Driven Trajectory Simplification (QDTS), which aims to simplify a trajectory database D to be within a storage budget such that its simplified database D' preserves the accuracy of query processing for *multiple* types of trajectory queries as much as possible when compared to the that on D.

Problem 1 (QDTS): Given a trajectory database D and a storage budget W indicating a fraction r of the original points in D that can be retained, **Query-Driven Trajectory Simplification** aims to find a trajectory database D' of simplified trajectories, such that the difference between query results on D and D' is minimized.

The QDTS problem relies on (1) a query type on a trajectory database and (2) a quality measure that captures the difference between the query results on the simplified database and those on the original database. To establish the former, we review the literature, including the evaluation papers on trajectory simplification [6], [7], [5] and a recent trajectory survey [38], and identify four widely-used queries, namely Range Query, kNN Query, Similarity Query, and Clustering. For the latter, we define query-based quality measures by following [6].

Range Query [39]. Given a trajectory database D, a range query with parameters $(q_{x_{min}}, q_{x_{max}}, q_{y_{min}}, q_{y_{max}}, q_{t_{min}}, q_{t_{max}})$ finds all trajectories that contain at least one point $p_i = (x_i, y_i, t_i)$ such that $q_{x_{min}} \leq x_i \leq q_{x_{max}}$, $q_{y_{min}} \leq y_i \leq q_{y_{max}}$, and $q_{t_{min}} \leq t_i \leq q_{t_{max}}$.

kNN Query [40]. Given a trajectory database D, a kNN query takes a query trajectory T_q and a time window $[t_s, t_e]$ as parameters and returns a set of k trajectories (denoted by R) such that $\forall T_i \in R, \forall T_j \in D - R$, $\Theta(T_q[t_s, t_e], T_i[t_s, t_e]) \leq \Theta(T_q[t_s, t_e], T_j[t_s, t_e])$, where $\Theta(\cdot, \cdot)$ represents a dissimilarity measure for trajectories. In this paper, we consider EDR [41] and t2vec [42] to instantiate $\Theta(\cdot, \cdot)$, as these represent non-learning and learning based trajectory similarity measures [38], [6], respectively. Note that our solution is orthogonal to the dissimilarity measure used.

Similarity Query [43]. Given a trajectory database D, a similarity query takes a trajectory T_q and a time window $[t_s, t_e]$ as inputs and returns a set of trajectories (denoted as R), such that $d(T_q[i], T_j[i]) \leq \delta$ for any $t_s \leq i \leq t_e$, where $T_j \in R$, $d(T_q[i], T_j[i])$ is the Euclidean distance between two points $T_q[i]$ and $T_j[i]$ and δ is a given distance threshold.

Trajectory Clustering [44]. Given a trajectory database, trajectory clustering partitions each trajectory into subtrajectories and then clusters subtrajectories based on some notion of distance among trajectories.

Quality Measures. We use the F_1-score for measuring the difference between query results on an original database D and those on a simplified database D'. The idea is to use the results on D as the ground truth and then measure the quality of the results on D' using the F_1-score. The smaller the difference between the query results is, the larger the F_1-score.

For range, similarity, and kNN queries, we denote by R_o and R_s the trajectory sets returned on D and D', respectively. We define precision (P), recall (R), and F_1-score as follows.

$$\mathrm{P} = \frac{|R_o \cap R_s|}{|R_s|} \quad \mathrm{R} = \frac{|R_o \cap R_s|}{|R_o|} \quad F_1 = 2 \cdot \frac{\mathrm{P} \cdot \mathrm{R}}{\mathrm{P} + \mathrm{R}} \tag{3}$$

In particular, for a kNN query, the precision, recall, and F_1-score are equal since $|R_o| = |R_s| = k$.

For the trajectory clustering query, we define R_o (resp. R_s) to be the set of pairs of trajectories, which are from the same cluster in the results on D (resp. D'), and then define the F_1-score as above.

QDTS v.s. Existing Trajectory Simplification Problems. The QDTS problem differs substantially from the existing error-driven trajectory simplification problems. First, it aims to optimize the data usability (i.e., the query accuracy) directly, as opposed to optimizing an error measure. Second, it targets a database of trajectories and simplifies the trajectories collectively, as opposed to separately.

Remarks. We emphasize that we only produce *one* simplified database and use the simplified database to support *multiple* types of queries including range query, kNN query, similarity query, clustering, and possibly others.

IV. METHODOLOGY

We propose a new algorithm called `RL4QDTS` for query accuracy driven trajectory simplification. It starts with the most simplified database, in which each simplified trajectory T' of an original trajectory T consists of only the first and last points of T. It then introduces original points into the simplified database iteratively until its budget is exhausted. For better efficiency, it builds an octree on the database of trajectories. Whenever it needs to choose a point, it first chooses a cube in the octree and then chooses a point in that cube. The octree recursively partitions a 2D spatial and 1D temporal space into 8 sub-spaces, which we call (spatial-temporal) cubes. This is essentially a sequential process of two decision tasks, and therefore, it adopts reinforcement learning (RL) since RL is widely known for its power of handling sequential decision processes. Specifically, `RL4QDTS` employs an agent (called Agent-Cube) to traverse the octree to find a cube. Then, it employs another agent (called Agent-Point) to choose a point in the chosen cube to be inserted into the simplified database. The decision making processes by the two agents are modeled as Markov decision processes (MDP) [16] and the MDPs are designed so that the agents cooperatively optimize the query accuracy on the simplified database.

We present the details of the MDPs of Agent-Cube and Agent-Point in Sections IV-A and IV-B, respectively. We then describe how the policies for the two MDPs are learned, in Section IV-C. We finally present the `RL4QDTS` algorithm that leverages the two agents for simplifying a trajectory database, in Section IV-D.

A. Agent-Cube: MDP for Choosing a Cube

Consider the task of choosing a cube. Agent-Cube chooses a cube by traversing the octree top-down, starting from the root node. Each time it visits a node, it decides whether to stop. If it stops, it means that the node's cube is chosen; otherwise, it decides which node among the 8 child nodes to visit. We define the Markov decision process (MDP) of Agent-Cube as follows.

(1) States. Let s^c denote a state of Agent-Cube's MDP, which we define as follows. We denote by $B_i^j (i > 1, 1 \leq j \leq 8)$ a cube of the octree, which is at the i^{th} level and corresponds to the j^{th} child node of its parent node. We designate B_1^1 to denote the cube of the root node. Consider that Agent-Cube is currently visiting cube B_i^j. For cube B_i^j, we use the number of trajectories (denoted by $M_{B_i^j}$) and the number of queries (denoted by $Q_{B_i^j}$) that fall into it, to capture the distributions of the data and queries. As queries are not available beforehand, we synthetically generate a workload of range queries, each query location is sampled randomly by following a certain distribution (e.g., data distribution). Formally, the state s^c at a cube B_i^j is defined by its 8 child nodes $(B_{i+1}^1, B_{i+1}^2, ..., B_{i+1}^8)$ with two distribution features (data and query) as follows.

$$s^c = \{\frac{M_{B_{i+1}^1}}{M_{B_i^j}}, \frac{Q_{B_{i+1}^1}}{Q_{B_i^j}}, ..., \frac{M_{B_{i+1}^8}}{M_{B_i^j}}, \frac{Q_{B_{i+1}^8}}{Q_{B_i^j}}\}, i \geq 1. \tag{4}$$

Here, the values of a state are normalized by dividing by the total numbers of trajectories and queries in cube B_i^j (i.e., $M_{B_i^j}$ and $Q_{B_i^j}$) to avoid data scale issues. We explain the intuition of the state design as follows. The data values in the states of cubes capture how trajectories are distributed over the cubes. For example, if a cube has only few trajectories and is sparse, an agent tends to select this cube to ensure that data in that cube is not lost. Similarly, query values in the cubes capture how queries are distributed over the cubes. Intuitively, an agent tends to select a cube with a larger value since the data would serve more queries. We note that in cases we have some knowledge of the query workload for testing (e.g., its distribution), we can generate query workloads by following the distribution for training; in cases we have no knowledge of the query workload for testing, we can generate a query workload by following the data distribution for training. In our experiments, we conduct experiments which verify to some extent the transferability of our method for cases where the query workload for testing does not follow that of the one used for training.

(2) Actions. Let a^c denote an action of Agent-Cube's MDP. With the currently visited cube being B_i^j, we define two possible types of action: (1) Proceed to visit one of the 8 child nodes, and (2) Stop the traversal and return the current

cube to Agent-Point, to choose a point within the chosen cube. Formally, a^c is defined as follows.

$$a^c = k \quad (1 \leq k \leq 9). \tag{5}$$

Here, $a^c = 1, 2, ..., 8$ means to traverse one of the 8 child nodes of the current one and $a^c = 9$ means to stop at the current node. Furthermore, we constrain the action space by only considering the cubes that involve trajectories. Suppose we take an action $a^c = k$, which corresponds to one of the two transition cases. Case 1: it explores the next cube B_{i+1}^k if $1 \leq k \leq 8$, and a new state at cube B_{i+1}^k can be computed using Equation 4. Case 2: it stops and returns to Agent-Point if $k = 9$. More details of Agent-Point are presented in Section IV-B.

(3) Rewards. When the action is to explore one of the 8 child nodes, the reward cannot be immediately observed, since no point has been inserted into the simplified database. When the action is to choose the current cube for Agent-Point to choose a point within the cube, the simplified database would be updated and some reward signal can be acquired (e.g., by measuring the difference between the query accuracy on the original database and that on the updated simplified database). In summary, Agent-Cube would finally choose a cube for Agent-Point and then acquire a certain reward signal. Therefore, we make Agent-Cube and Agent-Point share the same rewards, since they cooperate towards the same objective, i.e., learning a query-aware policy such that a simplified database preserves the query accuracy as much as possible compared to the original database. In particular, we set the reward of an action by Agent-Cube to be equal to that of the following action of choosing a point within the selected cube by Agent-Point. More details of the reward definition of Agent-Point are presented in Section IV-B.

B. Agent-Point: MDP for Choosing a Point

We denote the chosen cube by Agent-Cube as B for simplicity. Next, we define the MDP of Agent-Point for choosing a point within B to introduce to the database.

(1) States. Let s^p denote a state of Agent-Point's MDP, which we define as follows. Let N_B (resp. M_B) denote the number of points (resp. trajectories) in the cube B. To define the state, one idea is to incorporate all N_B points. However, this idea has two issues. (1) The definition in this way is N_B-dependent, which is not suitable for other cases when the number of points is not N_B. (2) N_B is generally very large. With this definition, the state space would be huge and the model is hard to train.

We design the states such that these two issues are avoided as follows. First, let $p_{s_a}^{T_i}$ and $p_{s_b}^{T_i}$ denote the first point and last point of a trajectory T_i within the cube, respectively. For each point $p_{s_j}^{T_i}$ in the cube with $s_a \leq s_j \leq s_b$, we define a pair of two values, denoted by $v(p_{s_j}^{T_i})$, as follows.

$$v(p_{s_j}^{T_i}) = (v_s(p_{s_j}^{T_i}), v_t(p_{s_j}^{T_i})). \tag{6}$$

The first value, denoted by $v_s(p_{s_j}^{T_i})$, is equal to the "spatial" distance between $p_{s_j}^{T_i}$ and the synchronous point on the segment linking the points immediately before and after $p_{s_j}^{T_i}$ in the trajectory T_i. The second value, denoted by $v_t(p_{s_j}^{T_i})$, is equal to the "temporal" difference between the time of $p_{s_j}^{T_i}$ and the time of $p_{s_j}^{T_i}$'s closest point on the segment linking the points immediately before and after $p_{s_j}^{T_i}$ in the trajectory T_i. The intuition of the two values is to capture the features of the point $p_{s_j}^{T_i}$ from both the spatial and temporal aspects given the context of trajectory simplification.

Among all points in each trajectory T_i, we then find a point (denoted as $p_{s_*}^{T_i}$) which has the maximum v_s, where s_* denotes its index. That is,

$$s_* = \arg\max_{s_a \leq s_j \leq s_b} v_s(p_{s_j}^{T_i}). \tag{7}$$

Finally, the state s^p of Agent-Point is defined as the set of K largest v_s values of $v(p_{s_*}^{T_i})$ among the B_M trajectories, that is

$$s^p = \{v(p_{s_*}^{T_{\pi(1)}}), v(p_{s_*}^{T_{\pi(2)}}), ..., v(p_{s_*}^{T_{\pi(K)}})\}, \tag{8}$$

where π denotes the permutation of $T_1, T_2, ..., T_{M_B}$ such that $v_s(p_{s_*}^{T_{\pi(1)}})$, $v_s(p_{s_*}^{T_{\pi(2)}})$, ..., $v_s(p_{s_*}^{T_{\pi(M_B)}})$ is sorted in a descending order. K ($K \leq M_B$) is a hyper-parameter that can be tuned empirically to control the size of the state space. Note that if a point has been introduced in the database, the point will not be used for the state definition.

Here, we refer to an example for illustrating the state definition. Consider a cube B_3^4 (the bottom right node at the third tree level) in Figure 2. It contains two points p_5 and p_8 for the definition. For p_5 (resp. p_8), we calculate the values as $(1.6, 0.5)$ (resp. $(1.3, 0.7)$) for capturing the spatial and temporal distances with respect to its simplified segment $\overline{p_4 p_6}$ on T_2 (resp. $\overline{p_7 p_9}$ on T_3). Then, the state is constructed as $s^p = \{(1.6, 0.5), (1.3, 0.7)\}$ with the setting of $K = 2$.

Our state design avoids the two aforementioned issues, where K is generally much smaller than N_B or M_B. With this design, a state has a fixed size that is independent from the number of trajectories in the cube.

(2) Actions. Let a^p denote an action of Agent-Point. The design of actions is consistent with the design of state s^p. Specifically, the actions are defined as follows:

$$a^p = k \quad (1 \leq k \leq K), \tag{9}$$

where action $a^p = k$ means to introduce point $p_{s_*}^{T_{\pi(k)}}$ into D'.

(3) Rewards. Since our objective is to obtain a simplified database that serves queries more effectively (i.e., minimizing the difference between the query results on the original database and those on the simplified database), the reward is expected to reflect the improvement of query performance as more points are included in the simplified database. To this end, we use the query workloads that have been used for defining the states (e.g., we use a set of range queries, where each query location is randomly sampled by following the data distribution). One option is to perform the queries after each point is inserted to the simplified database, which is associated with the transition from the current state s^p to the next state $s^{p'}$ when an action a^p is taken. However, it would be prohibitively costly to perform queries for each inserted

point. In addition, since the simplified database D' has not been fully constructed, the query improvement with inserting just one point is often negligible and it is hard to demonstrate the quality of the action.

In our design, we choose to perform the queries after Δ (e.g., $\Delta = 50$) points are inserted for achieving accumulative effects. Specifically, we denote the reward by R. At state s_i^p, we consider the simplified database (denoted by D'). At state $s_{i+\Delta}^p$, we consider the simplified database again (denoted by D''). We then define the reward R as follows.

$$R = \mathit{diff}(Q(D), Q(D')) - \mathit{diff}(Q(D), Q(D'')), \tag{10}$$

where $\mathit{diff}(Q(D), Q(D'))$ measures the difference between the results of queries on the original database D and the simplified database D'. The intuition is that if the difference for the simplified database D'' is smaller, then the reward is larger. Furthermore, we make the reward R be shared by all transitions that are involved when traversing from s_i^p to $s_{i+\Delta}^p$ as well as those of Agent-Cube that are involved in this process.

With the above reward definition, the objective of the MDP, i.e., maximizing the accumulative rewards, would be equivalent to that of the QDTS problem, i.e., minimizing the difference between queries on the original database and those on the simplified database. To see this, suppose we traverse a sequence of N' states $s_1^p, s_2^p, ..., s_{N'}^p$ (for simplicity, we assume $\Delta = 1$ for this analysis). Correspondingly, we receive a sequence of rewards $R_1, R_2, ..., R_{N'-1}$. We assume that the future rewards are accumulated without discounted rates, and thus the accumulative reward is calculated as follows.

$$\begin{aligned}\sum_{t=1}^{N'-1} R_t &= \sum_{t=1}^{N'-1} (\mathit{diff}(Q(D), Q(D'_t)) - \mathit{diff}(Q(D), Q(D''_t))) \\ &= \mathit{diff}(Q(D), Q(D'_1)) - \mathit{diff}(Q(D), Q(D''_{N'-1})) \\ &= C - \mathit{diff}(Q(D), Q(D''_{N'-1})),\end{aligned} \tag{11}$$

where D'_t (resp. D''_t) denotes the simplified database at the state s_t^p before (resp. after) the action a_t^p is performed. We regard the initial term $\mathit{diff}(Q(D), Q(D'_1))$ as a constant C and no points have been inserted at that state. Therefore, the objective of the MDP is to maximize $C - \mathit{diff}(Q(D), Q(D''_{N'-1}))$ or equivalently to minimize $\mathit{diff}(Q(D), Q(D''_{N'-1}))$, which is exactly the objective of QDTS.

C. Policy Learning via DQN

The core problem of a MDP is to find an optimal policy, which guides an agent to choose an action at a specific state, such that the accumulative reward is maximized. Considering that the states in our MDPs are continuous, we adopt the Deep-Q-Networks (DQN) [45] for learning a policy from the MDPs of Agent-Cube and Agent-Point. Specifically, we adopt the deep Q learning with replay memory [45] for learning the policy, denoted by $\pi_{\theta^c}(a|s^c)$ for Agent-Cube (resp. $\pi_{\theta^p}(a|s^p)$ for Agent-Point). The policy samples an action a at a given state s^c (resp. s^p) via DQN, whose parameters are denoted by θ^c (resp. θ^p). We note that other RL algorithms such as policy gradient can also be used for continuous state MDPs.

Algorithm 1: The framework of `RL4QDTS` algorithm

1 **Function** `RL4QDTS`($D = \langle T_1, T_2, ..., T_M \rangle$, W):
2 Build an octree OT for D;
3 **for** *i=1,2,...,M* **do**
4 Insert point $p_1^{T_i}$ and $p_{|T_i|}^{T_i}$ into D';
5 **end**
6 **for** *i=2M+1,2M+2,...,W* **do**
7 $B \leftarrow$ `Agent-Cube`(OT);
8 $D' \leftarrow$ `Agent-Point`(B, D');
9 **end**
10 **return** D'

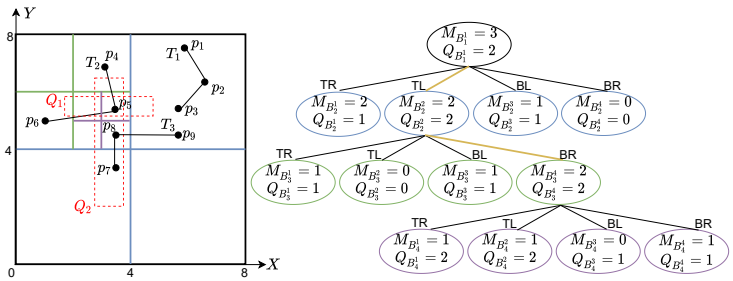
Algorithm 2: The `Agent-Cube`

1 **Function** `Agent-Cube`(OT):
2 $h \leftarrow 1$ and $l \leftarrow 1$;
3 **while** *true* **do**
4 Construct a state $s_h^c \leftarrow \{\frac{M_{B_{h+1}^1}}{M_{B_h^l}}, \frac{Q_{B_{h+1}^1}}{Q_{B_h^l}}, ..., \frac{M_{B_{h+1}^8}}{M_{B_h^l}}, \frac{Q_{B_{h+1}^8}}{Q_{B_h^l}}\}$;
5 Sample an action $a_h^c \sim \pi_{\theta^c}(a|s_h^c)$;
6 **if** $a_h^c = 9$ **then**
7 $B \leftarrow B_h^l$;
8 **Break;**
9 **else**
10 $h \leftarrow h + 1$ and $l \leftarrow a_h^c$;
11 **Continue**;
12 **end**
13 **end**
14 **return** B

D. The `RL4QDTS` Algorithm

Algorithm 1 details the framework of `RL4QDTS` with the learned policies of Agent-Cube and Agent-Point for the QDTS problem. Specifically, `RL4QDTS` starts by building an octree for the original trajectory database D (line 2), and then inserts the first and the last points of each trajectory into a simplified trajectory database D' (lines 3 – 5). The remaining budget $W - 2M$ is utilized in lines 6 – 9. First, it calls Agent-Cube (to be presented in Algorithm 2) to choose a cube B, and then the cube is fed into Agent-Point (to be presented in Algorithm 3) for updating D'. The process continues until the budget is exhausted. The `RL4QDTS` algorithm returns D', which contains W points (line 10).

Agent-Cube in Algorithm 2 first initializes the indexes h and l to indicate a cube B_h^l (line 2). To sample a cube, in lines 3 – 13, it constructs a state s_h^c using Equation 4 (line 4), and samples an action a_h^c with the learned policy $\pi_{\theta^c}(a|s_h^c)$, which takes s_h^c as input (line 5). If the action is $a_h^c = 9$, it breaks and returns the current cube denoted by B (lines 6 – 8); otherwise, it updates the indexes by $h \leftarrow h+1$ and $l \leftarrow a_h^c$, and explores the next cube B_h^l (lines 9 – 12).



Initial	Insert $p_1, p_3, p_4, p_6, p_7, p_9$ into D'		
Cube	Ag.	State	Action
B_1^1	C	$\{\frac{2}{3}, \frac{1}{2}, \frac{2}{3}, \frac{2}{2}, \frac{1}{3}, \frac{1}{2}, \frac{0}{3}, \frac{0}{2}\}$	Explore B_2^2
B_2^2	C	$\{\frac{1}{2}, \frac{1}{2}, \frac{0}{2}, \frac{0}{2}, \frac{1}{2}, \frac{1}{2}, \frac{2}{2}, \frac{2}{2}\}$	Explore B_3^4
B_3^4	C	$\{\frac{1}{2}, \frac{2}{2}, \frac{1}{2}, \frac{2}{2}, \frac{0}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\}$	Sample B_3^4
B_3^4	P	$\{(1.6, 0.5), (1.3, 0.7)\}$	Choose p_5
Output	Return $D' = \langle T_1', T_2', T_3' \rangle$, where $T_1' = \langle p_1, p_3 \rangle$, $T_2' = \langle p_4, p_5, p_6 \rangle$, $T_3' = \langle p_7, p_9 \rangle$		

Fig. 2: A running example of RL4QDTS, where we use a quadtree to simplify demonstration by eliminating the temporal dimension. **Left**: Three trajectories T_1, T_2, and T_3 exist in the original database D, along with two range queries Q_1 and Q_2 from the query workload. **Middle**: Quadtree nodes are labeled as TR, TL, BL, and BR (Top Right, Top Left, Bottom Left, and Bottom Right) with different colors representing quadtree levels. Agent-Cube traverses through nodes following yellow lines. **Right**: MDPs of Agent-Cube (C) and Agent-Point (P) for inserting point p_5 into the simplified database D'.

Algorithm 3: The `Agent-Point`

1 **Function** `Agent-Point`(B, D')**:**
2 Compute $v(p_{s_*}^{T_j})$ $(1 \le j \le M_B)$ for $T_j \in B$;
3 Maintains a descending permutation π of the values with a max-priority queue;
4 Construct a state $s^p \leftarrow \{v(p_{s_*}^{T_{\pi(1)}}), v(p_{s_*}^{T_{\pi(2)}}), ..., v(p_{s_*}^{T_{\pi(K)}})\}$;
5 Sample an action $a^p \sim \pi_{\theta^p}(a|s^p)$;
6 Insert the point $p_{s_*}^{T_{\pi(k)}}$ into D' where $a^p = k$ $(1 \le k \le K)$;
7 **return** D'

Agent-Point in Algorithm 3 takes a cube B as input and computes the value of $v(p_{s_*}^{T_j})$ for each trajectory T_j using Equations 6 and 7, where $1 \le j \le M_B$ (line 2). Then, it maintains a descending permutation π of the values with a max-priority queue (line 3). Next, it constructs a state s^p using Equation 8 (line 4), and samples an action with the learned policy $\pi_{\theta^p}(a|s^p)$, which takes s^p as input (line 5). Let $a^p = k$ $(1 \le k \le K)$ denote the sampled action. It then takes the action by inserting the point $p_{s_*}^{T_{\pi(k)}}$ into D' (line 6).

We illustrate the RL4QDTS Algorithm with the running example in Figure 2. Here, we use a quadtree (instead of an octree) by ignoring the temporal dimension of trajectory points for ease of demonstration. The input is a trajectory database $D = \langle T_1, T_2, T_3 \rangle$ with storage budget $W = 7$. Suppose the range query workload involves Q_1 and Q_2, where we have $Q_1(D) = \langle T_2 \rangle$ and $Q_2(D) = \langle T_2, T_3 \rangle$ based on the original database D. We build a quadtree and record the number of trajectories (M_B) and queries (Q_B) in the tree nodes. (1) The algorithm first inserts the first and the last points of each trajectory into D', meaning that the remaining budget is one point (i.e., 7-2*3=1). (2) Then, Agent-Cube starts at the root node B_1^1 and constructs its state by observing the four child nodes. It takes the action to explore node B_2^2 (Top Left in the figure). (3) Similarly, at B_2^2, it takes the action to explore node B_3^4 (Bottom Right). (4) At B_3^4, Agent-Cube receives the action of providing B_3^4 to Agent-Point. (5) Agent-Point constructs the state at cube B_3^4 and takes the action to insert point p_5 into D'. (6) Finally, the algorithm breaks from the loop and returns the simplified database D' since the budget is exhausted. We observe that D' outputs the same results (i.e., $Q_1(D') = \langle T_2 \rangle$ and $Q_2(D') = \langle T_2, T_3 \rangle$) as when querying D, which means that the query accuracy is preserved.

TABLE I: Dataset statistics.

Statistics	Geolife	T-Drive	Chengdu	OSM
# of trajectories	17,621	10,359	179,756	513,380
Total # of points	24,876,978	17,740,902	32,151,865	2,913,478,785
Ave. # of pts per traj	1,412	1,713	178	5,675
Sampling rate	1s $\sim$ 5s	177s	2s $\sim$ 4s	53.5s
Average length	9.96m	623m	25m	180m

In addition, we develop two techniques to enhance the effectiveness and efficiency of RL4QDTS. First, we constrain the octree traversal of Agent-Cube by a maximum tree depth E. If Agent-Cube reaches this level, it returns the currently visited cube to Agent-Point. The rationale is to prevent a very long traversal path for Agent-Cube, since in this case, it is difficult to train a policy to converge - recall that the reward is computed with delays. The benefits are verified in experiments. Second, we set a start level S so that the Agent-Cube starts traversing the octree by randomly sampling a cube following the query distribution (the one that has been used for defining states) from the start level S. The number of points in a cube decreases as the tree level increases. If Agent-Cube stops at the root level, Agent-Point will operate on all points in the database. Hyperparameter S can be used to avoid returning cells with excessive numbers of points.

Time complexity. The time complexity of the RL4QDTS algorithm is $O(N + W(n + \log M_B))$, where N, W, n, and M_B denote the total number of points in the original database, the storage budget, the maximum number of points in the input trajectories, and the maximum number of trajectories in the data cubes. Specifically, it takes $O(N)$ time to build an octree on the original database with maximum tree depth E, which is a small constant [46]. The part of processing of the remaining $W - 2M$ points dominates the complexity, including (1) choosing a cube by Agent-Cube with cost $O(1)$, which explores the octree for a bounded number of levels; (2) computing the values by Agent-Point with cost $O(n)$;

(3) maintaining the min-priority queue with cost $O(\log M_B)$; (4) constructing a state, sampling an action, and inserting a point by Agent-Point with cost $O(1)$ assuming K is a small constant. We note that the RL4QDTS algorithm has the same complexity as the error-driven algorithms [8], [9], [11] for simplifying a set of trajectories. In addition, we note that simplification is normally performed once offline, after which the simplified database is used for online querying.

Remarks. We train RL4QDTS with range queries only and then test it for different types of queries (including range query, kNN Query, Similarity Query, and Clustering) without retraining the model. The rationale behind our strategy is that the range query is a simple yet basic one and by training the model with range queries, the model would learn to capture essential spatial and temporal patterns of trajectories when simplifying them, which would then be useful for other types of queries. We follow this strategy and verify the transferability of our model among different types of queries in experiments.

V. EXPERIMENTS

A. Experimental Setup

Dataset. We conduct the experiments on four real-world trajectory datasets, Geolife [1], T-Drive [2], Chengdu [3] and OSM [4]. Geolife contains trajectories from 182 users during a period of five years (2007 – 2012), and the trajectory data is distributed across 30 cities in China, with most trajectories being from Beijing. T-Drive contains trajectories from 10,357 taxis when driving in Beijing over a period of one week. Chengdu contains taxi trajectories from 2016-11-01 to 2016-11-07, released by DiDi Chuxing. OSM is used to test the scalability, which contains three billion points, released by the community on OpenStreetMap. The datasets are widely used in previous trajectory simplification studies [6], [11], [10], and detailed statistics are shown in Table I.

Baselines. In the literature, no algorithms have been proposed for the QDTS problem. Given that the EDTS problem takes a storage budget for a trajectory as input, we consider existing algorithms that have been proposed for EDTS as baselines in our experiments. Specifically, we consider four algorithms, namely Top-Down [8], Bottom-Up [9], RLTS+ [11], and Span-Search [10]. Among them, Top-Down, Bottom-Up, and RLTS+ are general frameworks that can be applied with different error measures, while Span-Search works with DAD only. We adapt Top-Down, Bottom-Up, and RLTS+ in two ways. The first is to simplify each trajectory in the database one by one by calling one of the algorithms (this adaptation is denoted as "E"). The second is to consider the database as a whole and simplify the database by inserting or dropping points among all points in the database as it simplifies a trajectory (this adaptation is denoted as "W"). In summary, for each of the algorithms Top-Down, Bottom-Up, and RLTS+, we obtain 8 ($= 4 \cdot 2$) adaptations as baselines, each corresponding to a combination of an error measure SED, PED, DAD, or SAD, and an adaptation method ("E" and "W"). In total, we have 25 baselines including 24 ($= 3 \cdot 8$) adaptations of Top-Down, Bottom-Up, and RLTS+ and 1 adaption of Span-Search (we note that for Span-Search, the "W" adaptation is not possible).

Evaluation Platform. We implement RL4QDTS and the baselines in Python 3.6 and Keras 2.2.0. The experiments are conducted on a 10-cores server with an Intel(R) Core(TM) i9-9820X CPU @3.30GHz 64.0GB RAM and an Nvidia GeForce RTX 2080 GPU. The datasets and code are available via the link[5].

Model Training and Parameter Settings. We implement Agent-Cube with a two-layered feedforward neural network. The first layer has 25 neurons and uses the tanh activation function. The second layer has 9 neurons corresponding to the action space and uses a linear activation function. We set the hyperparameters S and E to be 9 and 12, respectively, based on empirical findings. We also implement Agent-Point with a two-layered feedforward neural network, where the first layer involves 25 neurons using the tanh activation function. The second layer involves K neurons corresponding to the action space and uses a linear activation function, where K is set to 2. We employ batch normalization in the neural networks to avoid data scale issues.

For training, we randomly sample 6,000 trajectories from Geolife (resp. 6,000 trajectories from T-Drive, 48,000 trajectories from Chengdu and 6,000 trajectories from OSM), and the remaining trajectories are used for testing. From the 6,000 (resp. 6,000, 48,000 and 6,000) trajectories, we randomly prepare 12 databases each with 500 (resp. 500, 4,000 and 500) trajectories. Further, we generate 5 episodes for each database for training the policy, and the best model is chosen during the training process. We note that the setting produces around one million transitions in the training process, which is sufficient to train a good policy with reasonable training time based on empirical findings. In addition, we set $\Delta = 50$, i.e., for every 50 points that have been inserted, we perform 100 range queries, each with a spatial region of 2km by 2km and a temporal duration of 7 days, for constructing states and acquiring rewards. Here, we only use the range query for training since it is a basic query type involving both the spatial and temporal dimensions. We vary the distributions of range queries across three distributions, namely (1) the data distribution, (2) the Gaussian distribution, and (3) the real distribution, for training the model. (1) and (2) are for the Geolife, T-Drive and OSM datasets and are adopted by following [47], where the parameters are set to be $\mu = 0.5$ and $\sigma = 0.25$ in the Gaussian distribution. (3) is for the Chengdu dataset, for which queries are generated near the pickup and dropoff locations that are provided in the dataset, and it may

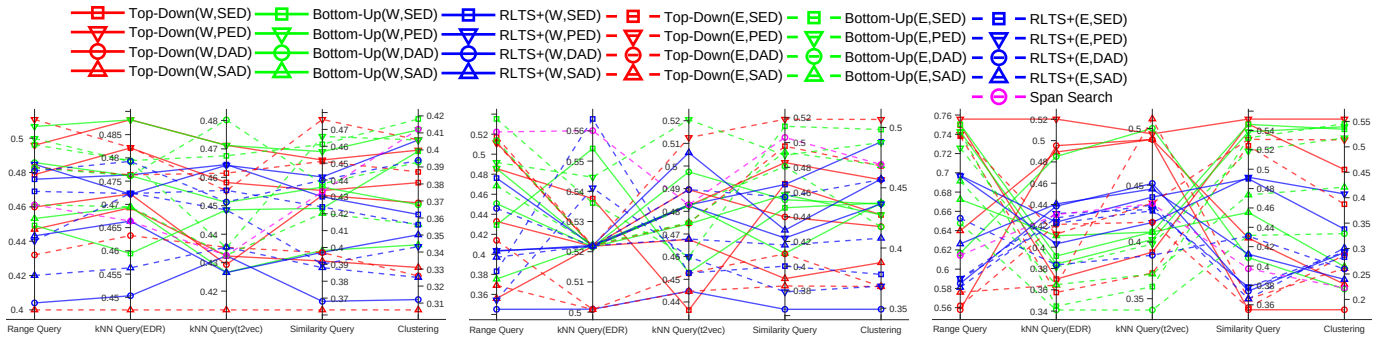
[1]https://www.microsoft.com/en-us/research/publication/geolife-gps-trajectory-dataset-user-guide/

[2]https://www.microsoft.com/en-us/research/publication/t-drive-trajectory-data-sample/

[3]https://drive.google.com/file/d/1onzDFpbD9OOfvOK7jHJ6Tpi2V4oKfxXR/view?usp=sharing

[4]https://star.cs.ucr.edu/?OSM/GPS\#center=43.6,-56.1\&zoom=2

[5]https://www.dropbox.com/sh/ui2yegn2wmok8hs/AAAXRfeKH1R7AHATmMmsW3Cga?dl=0



(a) Data distribution (b) Gaussian distribution (c) Real distribution

Fig. 3: Skyline selection with existing algorithms.

correspond to some real queries in ride-hailing services. The discount rate is set to 0.99. The RL4QDTS model is trained via Adam stochastic gradient descent with an initial learning rate of 0.01. The minimal ϵ is set to 0.1 with decay 0.99 for ϵ-greedy in DQN, and the size of the replay memory is set to 2000.

For testing, we notice the RL4QDTS involves some randomness of sampling a cube at the start level. Then, for each result of RL4QDTS, we run the algorithm 50 times and collect the averages and standard deviations of the query metrics. For range queries, we set the range query to be a cube with a spatial region of 2km by 2km and a temporal duration of 7 days. We use 7 days as the window length for both kNN and similarity queries. For kNN queries, we set $k = 3$ and use EDR and t2vec as the similarity measures. For EDR, we use a threshold of 2km and for t2vec, we adopt the settings as described in the original paper [42]. For similarity queries, the distance threshold is set to 5 km. For clustering, we adopt the TRACLUS algorithm by following the original paper [44].

B. Experimental Results

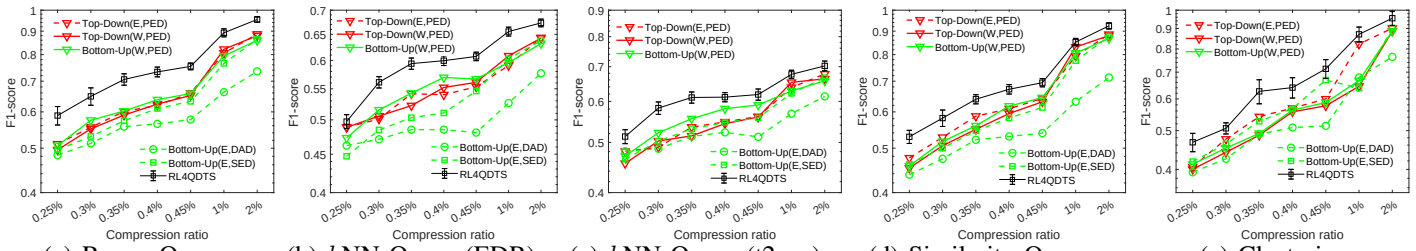
(1) Effectiveness evaluation (skyline selection of existing algorithms). Since we have 25 baselines, we select the skylines of the baselines for each query task to achieve more targeted comparisons. We construct a trajectory database D containing around 1.5 million points, and the storage budget for the simplification is set to $W = 0.25\% \cdot N$ for Geolife (resp. $W = 2\% \cdot N$ for Chengdu). In Figure 3, we show the effectiveness of the algorithms for five query tasks: range query, kNN Query(EDR), kNN Query(t2vec), Similarity Query, and clustering, with the three query distributions. For each task, we query 100 times and report the average results of the F_1-score as described in Section III-B. We conclude the selected baselines for comparisons as follows. For the data distribution, we observe that Top-Down(E,PED), Top-Down(W,PED), Bottom-Up(W,PED), Bottom-Up(E,DAD), and Bottom-Up(E,SED) are on the skyline. For the Gaussian distribution, we observe that Bottom-Up(E,SED), RLTS+(E,SED), Bottom-Up(E,PED), and Top-Down(E,PED) are on the skyline. For the real distribution, we observe that Top-Down(W,PED) and Top-Down(E,SAD) are on the skyline.

(2) Effectiveness evaluation (comparison with skyline). We compare RL4QDTS with the selected skyline methods for each query task. We vary the storage budget W from $0.25\% \cdot N$ to $2\% \cdot N$ for Geolife and T-Drive, and $2\% \cdot N$ to $20\% \cdot N$ for Chengdu. Here, a compression ratio of 0.25% means that we reduce the data by a factor of 400 (=100/0.25). That is, the lower the compression ratio is, the more the data is reduced. With the current settings of the compression ratio, (1) the data reduction rate is some 50-400 times for Geolife and T-Drive and 5-50 times for Chengdu, which looks reasonable in practice, and (2) the accuracy of the query processing is also acceptable (e.g., the F1 score is at least 60% in many cases as shown later on). We note that the trajectories in the Chengdu dataset are shorter than those in other datasets, as shown in Table I, and thus we set the budget higher for this Chengdu. Figure 4 shows the results on the two query distributions (i.e., data and Gaussian) on Geolife. For RL4QDTS, we show its error bars obtained by running the algorithm 50 times as described in Section V-A. The results based on T-Drive and Chengdu, shown in Figure 5 and Figure 6, respectively, demonstrate trends similar to those seen on Geolife. Overall, we observe that RL4QDTS consistently outperforms the existing error-driven methods across the different storage budgets, different query tasks with different generation distributions, and real datasets. Consider the results on Geolife for example. RL4QDTS outperforms the best skyline(s) by 34.6% (resp. 10.9%, 15.7%, 34.8%, and 39.9%) for range query (resp. kNN Query(EDR), kNN Query(t2vec), Similarity Query, and clustering) for the data distribution, and by 34.9% (resp. 11.6%, 14.5%, 27.6% and 22.8%) for range query (resp. kNN Query(EDR), kNN Query(t2vec), Similarity Query and clustering) for the Gaussian distribution. This is because RL4QDTS takes the query quality as the objective for trajectory simplification, and learns a query accuracy aware policy for the simplification to preserve the query quality directly; while existing methods aim to minimize a given error measure, and query quality is no considered directly.

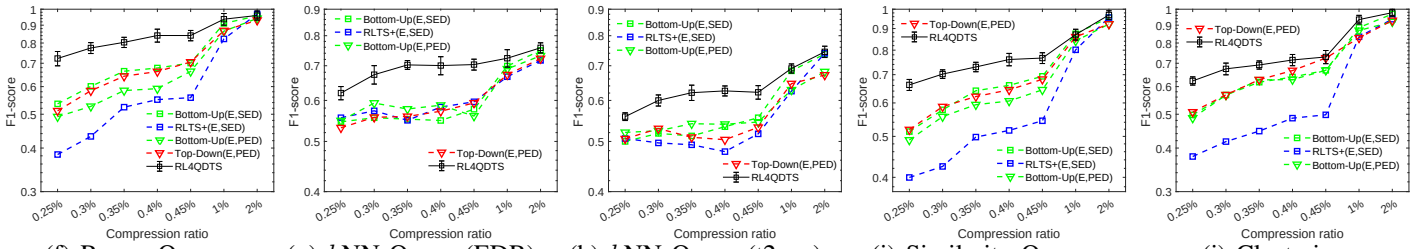
TABLE II: Ablation study for RL4QDTS (Geolife).

Effectiveness	Range Query	Time (s)
RL4QDTS	$\mathbf{0.733 \pm 0.018}$	61.11
w/o Agent-Cube	0.673 ± 0.023	50.32
w/o Agent-Point	0.716 ± 0.021	59.31
w/o Agent-Cube and Agent-Point	0.641 ± 0.023	48.18

(3) Effectiveness evaluation (ablation study). We conduct an ablation study to investigate the effects of Agent-Cube and

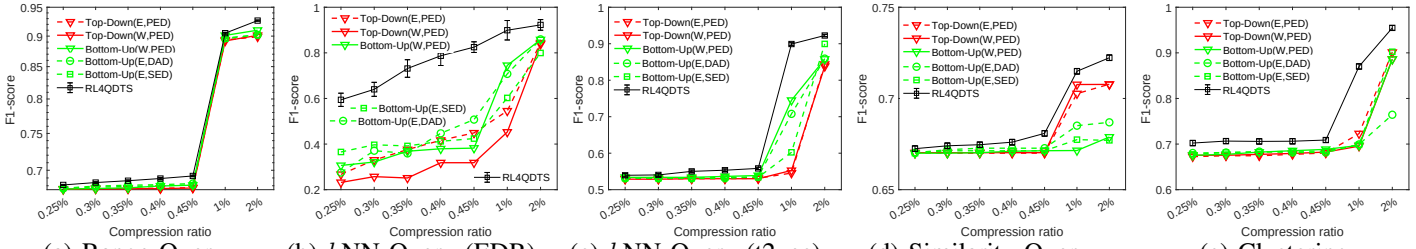


(a) Range Query (b) kNN Query (EDR) (c) kNN Query (t2vec) (d) Similarity Query (e) Clustering

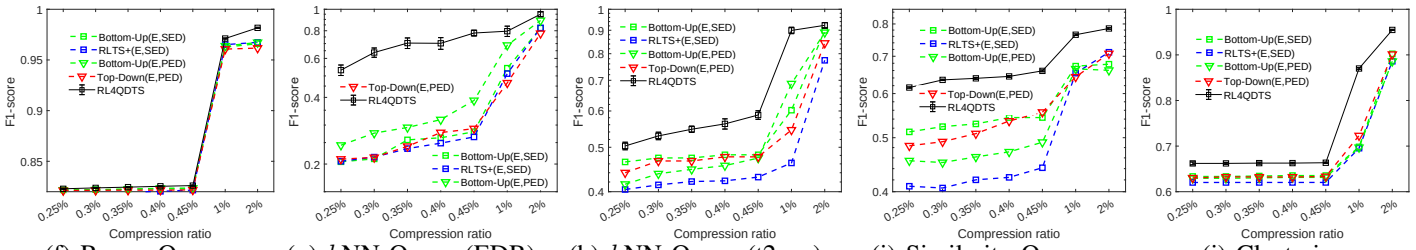


(f) Range Query (g) kNN Query (EDR) (h) kNN Query (t2vec) (i) Similarity Query (j) Clustering

Fig. 4: Comparison with skylines on Geolife (data distribution (a)-(e) and Gaussian distribution (f)-(j)).



(a) Range Query (b) kNN Query (EDR) (c) kNN Query (t2vec) (d) Similarity Query (e) Clustering



(f) Range Query (g) kNN Query (EDR) (h) kNN Query (t2vec) (i) Similarity Query (j) Clustering

Fig. 5: Comparison with skylines on T-Drive (data distribution (a)-(e) and Gaussian distribution (f)-(j)).

TABLE III: Impacts of parameter S for RL4QDTS (Geolife), where RQ denotes the Range Query.

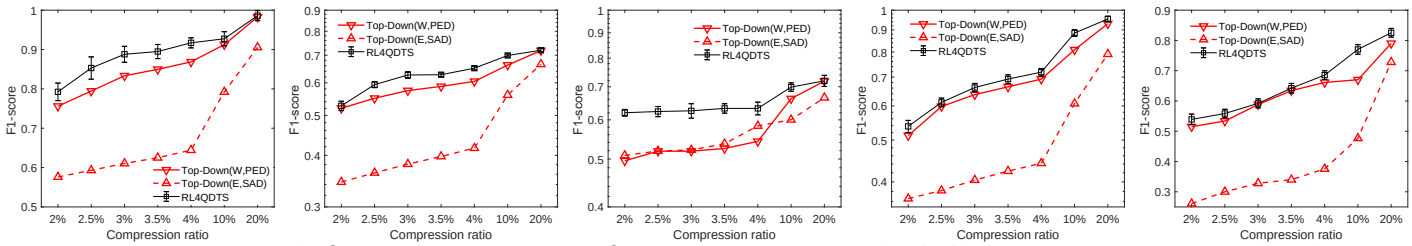
Start	7	8	9	10	11
RQ	0.557 ± 0.018	0.608 ± 0.019	$\mathbf{0.733 \pm 0.018}$	0.712 ± 0.021	0.686 ± 0.017
Time(s)	74.37	63.28	61.11	57.76	51.83

TABLE IV: Impacts of parameter E for RL4QDTS (Geolife).

End	10	11	12	13	14
RQ	0.673 ± 0.023	0.681 ± 0.018	$\mathbf{0.733 \pm 0.018}$	0.693 ± 0.019	0.671 ± 0.021
Time(s)	50.32	57.76	61.11	62.31	67.42

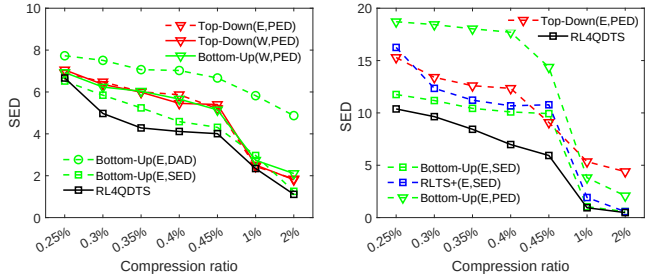
Agent-Point in RL4QDTS. (1) We drop Agent-Cube by setting the start level $S = 9$ and the end level $E = 10$, so that Agent-Cube reduces to randomly sampling a cube according to the data distribution and then returning the cube to Agent-Point. (2) We drop Agent-Point and instead insert the point with the maximum value into a simplified database. (3) We drop both Agent-Cube and Agent-Point with the strategies described above. Table II reports the average results of 100 range queries with a distribution that follows the data distribution on a randomly sampled trajectory database with around 1.5 million points from Geolife. Overall, all components contribute to the result. Specifically, we observe that Agent-Cube improves the effectiveness by 6.1% and Agent-Point improves the effectiveness by 2.4%. As expected, the two agents cooperate to optimize query quality. In addition, we notice that without Agent-Cube, Agent-Point, or both, the efficiency improves since the agents employ learning models (i.e., deep neural networks) to make decisions. Without them, simple operations are used for the same tasks.

(4) Effectiveness evaluation (deformation study). We inves-



(a) Range Query (b) kNN Query (EDR) (c) kNN Query (t2vec) (d) Similarity Query (e) Clustering

Fig. 6: Comparison with skylines on Chengdu (real distribution (a)-(e)).



(a) Data distribution Gaussian distribution

Fig. 7: Deformation study (SED errors of trajectories returned by queries)

tigate the deformation of the trajectories returned by queries. In Figure 7, we run 100 range queries with the data and Gaussian distributions, and report the average SED of the returned trajectories, which measures the deformation in terms of the synchronized Euclidean distances (SED) between the original trajectories and their simplified ones. As expected, the deformation of RL4QDTS is consistently lower than skyline methods. This is because RL4QDTS is a query-aware solution, which preserves more points for those trajectories to answer the queries. For the skyline methods, they fail to preserve the trajectories returned by queries, though the distances (e.g., SED) can be optimized explicitly for all trajectories including both the returned ones and others.

(5) Parameter study (varying parameter S in Agent-Cube). We evaluate the effect of the start level S in Agent-Cube. We report the average F_1-score of 100 range queries that follows the data distribution on a database with around 1.5 million points, and we report the running time. Here, we fix the end level $E = 12$ and vary the start level S from 7 to 11. In Table III, we observe that the efficiency improves as S increases. This is because the efficiency of RL4QDTS depends on the number of points in a sampled cube. A larger start level S corresponds to a smaller cube containing fewer candidate points, making the model run more efficiently to select a point. In contrast, a larger cube (corresponding to a smaller start level S) contains more candidates, e.g., in the extreme case of setting $S = 1$, the Agent-Cube may select a point from the whole database, resulting in scalability issue. In addition, we observe that a moderate setting (i.e., $S = 9$) brings the best effectiveness. The reason is that a smaller cube would make the model miss potential points to be introduced. Further, while a larger cube would contain many candidates of points, the sampled cube may not capture the query distribution accurately, e.g., in the extreme case that a cube covers the whole database, it does not reflect how queries are distributed at a finer granularity.

(6) Parameter study (varying parameter E in Agent-Cube). We study the effect of end level E and fix the start level $S = 9$. We consider the effectiveness by running 100 range queries with the data distribution on a database with around 1.5 million points. We report the average F_1-score and the running time in Table IV. We observe that the effectiveness improves as the end level E increases and then degrades as E increases further. This is because parameter E controls the maximum depth of the octree traversal. When E is small, Agent-Cube has a very limited search space, and the selected cubes may not be of good quality. When E is large, it is more difficult to train the RL policy to converge. In addition, the model runs fast with a smaller E, since it makes the model stops earlier. We choose $E = 12$ as it brings the best usability and runs comparably fast.

(7) Parameter study (varying parameter K in Agent-Point). We study the effect of parameter K, which controls the state space of Agent-Point for decision-making. We construct a database with around 1.5 million points randomly sampled from Geolife, and report the average results of 100 range queries with the data distribution and the corresponding running time. In Table V, we observe that the setting of $K = 2$ gives the best effectiveness, and runs reasonably fast. When $K = 1$, it reduces to greedily choose the point with the maximum value within a cube. When K becomes larger, the model performance degrades, since it becomes more difficult to train the model to choose a point among a larger number of candidate points. In addition, the time cost increases as K increases because more time is spent on constructing states. We set K to 2 by default, since this provides a reasonable trade-off between effectiveness and efficiency.

(8) Parameter study (varying parameter k in kNN query). We study the effect of different k in kNN queries with EDR and t2vec on Geolife. In Table VI, we vary the k from 1 to 5, and report the effectiveness by running 100 kNN queries with the data distribution. We observe that the effectiveness improves as k increases, because it overlaps more similar trajectories with a larger k in the returned queries. In addition, the results on EDR and t2vec show similar trends.

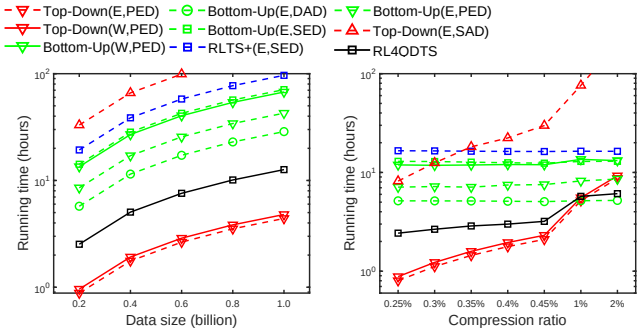
(9) Scalability test (varying the data size N). We study scalability when varying the database size on OSM. We

TABLE V: Impacts of parameter K for RL4QDTS (Geolife).

K	1	2	3	4	5
RQ	0.716 ± 0.021	$\mathbf{0.733 \pm 0.018}$	0.724 ± 0.021	0.723 ± 0.023	0.706 ± 0.023
Time(s)	59.31	61.11	62.43	63.85	64.35

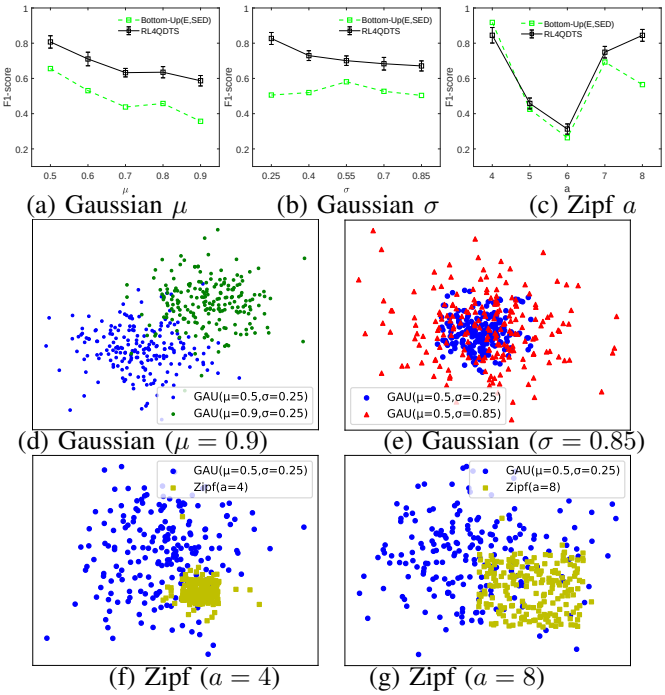
TABLE VI: Impacts of k in kNN query for RL4QDTS (Geolife).

kNN	1	2	3	4	5
EDR	0.281 ± 0.013	0.472 ± 0.011	0.594 ± 0.013	0.641 ± 0.015	0.669 ± 0.015
t2vec	0.372 ± 0.016	0.525 ± 0.016	0.611 ± 0.015	0.661 ± 0.013	0.686 ± 0.015



(a) Varying data size (b) Varying budget size

Fig. 8: Efficiency evaluation (OSM)



(a) Gaussian μ (b) Gaussian σ (c) Zipf a

(d) Gaussian ($\mu = 0.9$) (e) Gaussian ($\sigma = 0.85$)

(f) Zipf ($a = 4$) (g) Zipf ($a = 8$)

Fig. 9: Transferability test

compare all skyline methods as shown in Figure 3, and vary the trajectory database size N from 0.2 billion to 1 billion points, with a fixed storage budget $W = 0.25\% \cdot N$. The running times (the maximum is set to 100 hours) are shown in Figure 8(a). Overall, we observe that RL4QDTS runs faster than many of the existing methods. It is only slower than the methods adapted from Top-Down because these methods are based on the idea of iteratively introducing points from the original single trajectory or whole database according to some simple criterion that can be computed efficiently. In contrast, RL4QDTS employs a learned policy for that task to improve effectiveness, and this incurs time costs for constructing states and sampling actions of the two agents. Besides, we notice that the algorithms adapted with the "W" option are generally slower than the algorithms adapted with the "E" option. For example, Bottom-Up(E,PED) is faster than Bottom-Up(W,PED) by around 40%, since the Bottom-Up(W,PED) operates on all points in the database, which generally is more expensive. The results are qualitatively similar on other datasets and omitted.

(10) Efficiency evaluation (varying the budget size W). We further study the effect of budget size W from $0.25\% \cdot N$ to $2\% \cdot N$, with a fixed N of 0.1 billion points. Figure 8(b) illustrates the running time on Geolife. RL4QDTS is slower than the Top-Down adaptions, but is faster than the Bottom-Up adaptions by at least a factor of two times. As W increases, RL4QDTS becomes faster than Top-Down adaptions, because it computes the values based on a partial trajectory within a cube by Agent-Point; however, Top-Down adaptions computes that values based on a whole trajectory. In addition, the running times of the Top-Down adaptions increase, while those of the Bottom-Up adaptions decrease slightly as W increases, which may be explained by their strategies for conducting simplification (i.e., inserting vs dropping points), and this is consistent with their time complexities. Again, algorithms with the "W" is slower than that with the "E". The results are qualitatively similar on other datasets and omitted.

(11) Training time. In Table VII, we show the training time of RL4QDTS on Geolife with the default settings described in Section V-A. We randomly construct 5 training sets with 3,000 to 7,000 trajectories. For each training set, we report the time cost (hours) and effectiveness based on the selected best model during the training. We observe that the training generally takes several minutes and that the cost increases almost linearly with the number of trajectories. The effectiveness improves slightly when the number of trajectories exceeds 6,000. We use the setting of 6,000 trajectories to train the RL4QDTS model, which is enough to obtain a good model with a reasonable training cost. We further study the impacts of Δ in Table VIII. We observe that the training time decreases as Δ increases, because it performs fewer queries for obtaining rewards. Besides, a moderate Δ is with the best effectiveness. This is because with a smaller Δ, it is hard to demonstrate the quality of actions, and with a larger Δ, it causes a long delayed reward during the training.

(12) Transferability test (with distribution changes). We consider two scenarios to test RL4QDTS transferability. First,

TABLE VII: Training cost (Geolife).

Size	3,000	4,000	5,000	6,000	7,000
RQ	0.703 ± 0.017	0.714 ± 0.015	0.724 ± 0.021	$\mathbf{0.733 \pm 0.018}$	0.733 ± 0.018
Time(h)	0.67	0.88	1.15	1.36	1.55

TABLE VIII: Impacts of Δ for training (Geolife).

Δ	20	30	40	50	60
RQ	0.681 ± 0.017	0.686 ± 0.016	0.701 ± 0.015	$\mathbf{0.733 \pm 0.018}$	0.685 ± 0.018
Time(h)	2.35	1.91	1.44	1.36	1.34

we train RL4QDTS based on a query workload of range queries following the Gaussian distribution with $\mu = 0.5$ and $\sigma = 0.25$ on Geolife, and test its effectiveness for range queries which follow the Gaussian distributions with different μ (from 0.5 to 0.9), and σ (from 0.25 to 0.85). The results are shown in Figure 9(a) and (b). This is for testing the transferability for moderate changes in query distribution. Second, we use the same model that has been trained based on the Gaussian distribution and test its effectiveness for range queries which follow a Zipf distribution with different exponent parameters a (from 4 to 8). The a controls the density of the queries. We empirically set a starting from 4, since it will produce many duplicated queries that are closely together when a is further smaller. The results are shown in Figure 9(c). This is for testing the transferability for significant changes in query distribution. We also report the results of the best baseline Bottom-Up(E,SED). In Figure 9(a) and (b), we observe that as the change becomes more significant, RL4QDTS has its performance degrades slightly yet it still consistently outperforms the baseline across all settings of μ and σ. In Figure 9(c), we observe that RL4QDTS performs comparably well with the baseline in most cases and better sometimes though the query workload distribution has changed drastically, which shows the robustness of our method. This may be explained by that RL4QDTS does not rely on any error measures and preserves patterns and knowledge embedded in the data via neural networks, which remain useful to optimize queries even if the distributions are changed. We further visualize the distribution changes in Figure 9(d)-(g).

VI. CONCLUSION

We introduce the query accuracy-driven trajectory simplification problem, aiming to minimize the difference between query results on the original database and on a simplified one. Our novel solution, RL4QDTS, employs multi-agent reinforcement learning, enabling collective trajectory simplification while directly optimizing the QDTS problem's objective. Extensive experiments on real-world trajectory datasets demonstrate that RL4QDTS consistently outperforms existing EDTS algorithms in five query processing operations. One promising research direction is to explore data-driven approaches for compressing road network-based trajectories to enhance effectiveness and efficiency.

REFERENCES

[1] Z. Chen, H. T. Shen, and X. Zhou, "Discovering popular routes from trajectories," in *ICDE*, 2011, pp. 900–911.

[2] Z. Li, J. Han, M. Ji, L.-A. Tang, Y. Yu, B. Ding, J.-G. Lee, and R. Kays, "Movemine: Mining moving object data for discovery of animal movement patterns," *TIST*, vol. 2, no. 4, pp. 1–32, 2011.

[3] Z. Wang, C. Long, G. Cong, and C. Ju, "Effective and efficient sports play retrieval with deep representation learning," in *SIGKDD*, 2019, pp. 499–509.

[4] Z. Wang, C. Long, G. Cong, and Q. Zhang, "Error-bounded online trajectory simplification with multi-agent reinforcement learning," in *KDD*, 2021, pp. 1758–1768.

[5] X. Lin, S. Ma, J. Jiang, Y. Hou, and T. Wo, "Error bounded line simplification algorithms for trajectory compression: An experimental evaluation," *TODS*, vol. 46, no. 3, pp. 1–44, 2021.

[6] D. Zhang, M. Ding, D. Yang, Y. Liu, J. Fan, and H. T. Shen, "Trajectory simplification: an experimental study and quality analysis," *PVLDB*, vol. 11, no. 9, pp. 934–946, 2018.

[7] H. Cao, O. Wolfson, and G. Trajcevski, "Spatio-temporal data reduction with deterministic error bounds," *The VLDB Journal*, vol. 15, no. 3, pp. 211–228, 2006.

[8] J. E. Hershberger and J. Snoeyink, *Speeding up the Douglas-Peucker line-simplification algorithm*. University of British Columbia, Department of Computer Science, 1992.

[9] E. Keogh, S. Chu, D. Hart, and M. Pazzani, "An online algorithm for segmenting time series," in *ICDM*, 2001, pp. 289–296.

[10] C. Long, R. C.-W. Wong, and H. Jagadish, "Trajectory simplification: on minimizing the direction-based error," *PVLDB*, vol. 8, no. 1, pp. 49–60, 2014.

[11] Z. Wang, C. Long, and G. Cong, "Trajectory simplification with reinforcement learning," in *ICDE*, 2021, pp. 684–695.

[12] M. Potamias, K. Patroumpas, and T. Sellis, "Sampling trajectory streams with spatiotemporal criteria," in *SSDBM*, 2006, pp. 275–284.

[13] J. Muckell, J.-H. Hwang, V. Patil, C. T. Lawson, F. Ping, and S. Ravi, "Squish: an online approach for gps trajectory compression," in *Computing for Geospatial Research & Applications*, 2011, pp. 1–8.

[14] J. Muckell, P. W. Olsen, J.-H. Hwang, C. T. Lawson, and S. Ravi, "Compression of trajectory data: a comprehensive evaluation and new approach," *GeoInformatica*, vol. 18, no. 3, pp. 435–460, 2014.

[15] J. L. Bentley, "Multidimensional binary search trees used for associative searching," *Communications of the ACM*, vol. 18, no. 9, pp. 509–517, 1975.

[16] R. S. Sutton and A. G. Barto, *Reinforcement learning: An introduction*. MIT press, 2018.

[17] D. H. Douglas and T. K. Peucker, "Algorithms for the reduction of the number of points required to represent a digitized line or its caricature," *Cartographica*, vol. 10, no. 2, pp. 112–122, 1973.

[18] X. Lin, S. Ma, H. Zhang, T. Wo, and J. Huai, "One-pass error bounded trajectory simplification," *PVLDB*, vol. 10, no. 7, pp. 841–852, 2017.

[19] X. Lin, J. Jiang, S. Ma, Y. Zuo, and C. Hu, "One-pass trajectory simplification using the synchronous euclidean distance," *VLDBJ*, vol. 28, no. 6, pp. 897–921, 2019.

[20] J. Liu, K. Zhao, P. Sommer, S. Shang, B. Kusy, and R. Jurdak, "Bounded quadrant system: Error-bounded trajectory compression on the go," in *ICDE*, 2015, pp. 987–998.

[21] N. Meratnia and A. Rolf, "Spatiotemporal compression techniques for moving point objects," in *EDBT*, 2004, pp. 765–782.

[22] W. S. Chan and F. Chin, "Approximation of polygonal curves with minimum number of line segments or minimum error," *International Journal of Computational Geometry & Applications*, vol. 6, no. 01, pp. 59–77, 1996.

[23] C. Long, R. C.-W. Wong, and H. Jagadish, "Direction-preserving trajectory simplification," *PVLDB*, vol. 6, no. 10, pp. 949–960, 2013.

[24] S. Wang and H. Ferhatosmanoglu, "Ppq-trajectory: spatio-temporal quantization for querying in large trajectory repositories," *PVLDB*, vol. 14, no. 2, pp. 215–227, 2020.

[25] M. van de Kerkhof, I. Kostitsyna, M. Löffler, M. Mirzanezhad, and C. Wenk, "Global curve simplification," *arXiv preprint arXiv:1809.10269*, 2018.

[26] Y. Han, W. Sun, and B. Zheng, "Compress: A comprehensive framework of trajectory compression in road networks," *TODS*, vol. 42, no. 2, pp. 1–49, 2017.

[27] R. Song, W. Sun, B. Zheng, and Y. Zheng, "Press: A novel framework of trajectory compression in road networks," *arXiv preprint arXiv:1402.1546*, 2014.

[28] T. Li, R. Huang, L. Chen, C. S. Jensen, and T. B. Pedersen, "Compression of uncertain trajectories in road networks," *PVLDB*, vol. 13, no. 7, pp. 1050–1063, 2020.

[29] T. Li, L. Chen, C. S. Jensen, and T. B. Pedersen, "Trace: real-time compression of streaming trajectories in road networks," *Proceedings of the VLDB Endowment*, vol. 14, no. 7, pp. 1175–1187, 2021.

[30] D. A. Huffman, "A method for the construction of minimum-redundancy codes," *Proceedings of the IRE*, vol. 40, no. 9, pp. 1098–1101, 1952.

[31] Z. Wang, C. Long, G. Cong, and Y. Liu, "Efficient and effective similar subtrajectory search with deep reinforcement learning," *PVLDB*, vol. 13, no. 12, pp. 2312–2325, 2020.

[32] Z. Yang, B. Chandramouli, C. Wang, J. Gehrke, Y. Li, U. F. Minhas, P.-Å. Larson, D. Kossmann, and R. Acharya, "Qd-tree: Learning data layouts for big data analytics," in *SIGMOD*, 2020, pp. 193–208.

[33] K. Lin, R. Zhao, Z. Xu, and J. Zhou, "Efficient large-scale fleet management via multi-agent deep reinforcement learning," in *SIGKDD*, 2018, pp. 1774–1783.

[34] J. Liu, K. Zhao, P. Sommer, S. Shang, B. Kusy, J.-G. Lee, and R. Jurdak, "A novel framework for online amnesic trajectory compression in resource-constrained environments," *TKDE*, vol. 28, no. 11, pp. 2827–2841, 2016.

[35] R. Bellman, "On the approximation of curves by line segments using dynamic programming," *Communications of the ACM*, vol. 4, no. 6, p. 284, 1961.

[36] B. Ke, J. Shao, Y. Zhang, D. Zhang, and Y. Yang, "An online approach for direction-based trajectory compression with error bound guarantee," in *APWeb*, 2016, pp. 79–91.

[37] B. Ke, J. Shao, and D. Zhang, "An efficient online approach for direction-preserving trajectory simplification with interval bounds," in *MDM*, 2017, pp. 50–55.

[38] S. Wang, Z. Bao, J. S. Culpepper, and G. Cong, "A survey on trajectory data management, analytics, and learning," *CSUR*, vol. 54, no. 2, pp. 1–36, 2021.

[39] H. Cao, O. Wolfson, and G. Trajcevski, "Spatio-temporal data reduction with deterministic error bounds," in *Proceedings of the 2003 joint workshop on Foundations of mobile computing*, 2003, pp. 33–42.

[40] E. Frentzos, K. Gratsias, N. Pelekis, and Y. Theodoridis, "Nearest neighbor search on moving object trajectories," in *International Symposium on Spatial and Temporal Databases*. Springer, 2005, pp. 328–345.

[41] L. Chen, M. T. Özsu, and V. Oria, "Robust and fast similarity search for moving object trajectories," in *SIGMOD*, 2005, pp. 491–502.

[42] X. Li, K. Zhao, G. Cong, C. S. Jensen, and W. Wei, "Deep representation learning for trajectory similarity computation," in *ICDE*, 2018, pp. 617–628.

[43] Y. Chen and J. M. Patel, "Design and evaluation of trajectory join algorithms," in *Proceedings of the 17th ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems*, 2009, pp. 266–275.

[44] J.-G. Lee, J. Han, and K.-Y. Whang, "Trajectory clustering: a partition-and-group framework," in *SIGMOD*, 2007, pp. 593–604.

[45] V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, and M. Riedmiller, "Playing atari with deep reinforcement learning," *arXiv preprint arXiv:1312.5602*, 2013.

[46] H. Samet, "The quadtree and related hierarchical data structures," *ACM Computing Surveys (CSUR)*, vol. 16, no. 2, pp. 187–260, 1984.

[47] J. Qi, G. Liu, C. S. Jensen, and L. Kulik, "Effectively learning spatial indices," *Proceedings of the VLDB Endowment*, vol. 13, no. 12, pp. 2341–2354, 2020.