Trajectory Simplification with Reinforcement Learning

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***Abstract*—Trajectory data is used in various applications including traffic analysis, logistics, and mobility services. It is usually collected continuously by sensors and accumulated at a server resulting in big volume. A common practice is to conduct trajectory simplification which is to drop some points of a trajectory when they are being collected (online mode) and/or after they are accumulated (batch mode). Existing algorithms usually involve some decision making tasks (e.g., deciding which point to drop), for which, some human-crafted rules are used. In this paper, we propose to learn a policy for the decision making tasks via reinforcement learning (RL) and develop trajectory simplification methods based on the learned policy. Compared with existing algorithms, our RL-based methods are data-driven and can adapt to different dynamics underlying the problem. We conduct extensive experiments to verify that our RL-based methods compute simplified trajectories with smaller errors while running comparably fast (and faster in the batch mode) compared with existing methods.**

***Index Terms*—trajectory data, trajectory simplification, reinforcement learning**

I. INTRODUCTION

Trajectory data is a data type that captures traces of moving objects such as vehicles, pedestrians, robots, etc. It is central to many applications such as urban mobility analysis, logistics, transportation, sports games [1], etc. Trajectory data is typically generated continuously and collected by remote sensors such as GPS devices. One typical scenario is that a sensor periodically checks the coordinates and time, which corresponds to a time-stamped location (called spatio-temporal point or simply point), and stores the point in a buffer. Typically, a sensor has a small storage budget, low computation capability, and limited network bandwidth. A consequent issue is that the buffer would become occupied frequently and the workload of transmitting the points is high. In addition, in some applications, there could be hundreds of thousands of sensors, which collect trajectory data simultaneously. Once the trajectory data collected by all these sensors is accumulated at a server, the volume would be huge, which has been illustrated by several existing studies (see the survey paper [2]). A consequent issue is that the huge volume of trajectory data would increase the storage cost and more importantly make the query processing on the data expensive.

A common practice that has been used to deal with the aforementioned two issues is to conduct *trajectory simplification*, which essentially is to drop some points of a given trajectory and keep the remaining ones as a simplified trajectory. Specifically, in the online mode, the trajectory data is inputted point by point and once a point is dropped, it is no longer accessible. In the batch mode, the trajectory data is inputted completely once and remains accessible during the whole course of trajectory simplification. The rationale behind trajectory simplification is two-fold. First, not all points of a trajectory carry equal amount of information and some carry little or even no information. For example, when an object moves along a straight line at a constant speed, all points except for the first and last ones carry no information and could be dropped. Second, with some point dropped, the burden on the transmission, storage, and query processing would be lowered down significantly. In this paper, we consider a problem of trajectory simplification, which is to drop at least a certain number of points (or equivalently to keep at most a certain number of points) such that the information loss, captured as the "error" of the simplified trajectory, is minimized. We call this problem *Min-Error*.

Quite a few algorithms exist for the Min-Error problem in the online mode, including STTrace [3], SQUISH [4], and SUISH-E [5]. All these algorithms share the idea that it maintains a buffer of a certain size and keeps storing points in the buffer, and whenever the buffer becomes full, it picks one point from the buffer and drops it. In addition, they all make the decision on which point to drop by defining some "importance value" of each point and always dropping the point with the least importance value. Roughly, the importance value of a point is defined as some form of error that would be introduced when the point is dropped since a smaller error means that the point is less important and should be dropped. When a point is dropped, the importance values of the remaining points should be updated, in which the existing algorithms differ from one another. STTrace simply re-computes the values, while SQUISH distributes the value of a point that has been dropped to its neighboring points (so that the importance values would be carried on) and SQUISH-E is similar to SQUISH with only some slight refinements on the update procedure. As could be noticed, these algorithms are mainly based on some human-crafted rules for deciding which point to drop.

In this paper, we propose a reinforcement learning (RL) method for the trajectory simplification problem in the online mode. We treat the trajectory simplification problem (in the online mode) as one of a sequential decision process, i.e., it scans a trajectory sequentially, and whenever the buffer is full, it makes a decision on which point to drop. We then model the

process as a *Markov decision process* (MDP), learn the policy for the MDP via a widely-used *policy gradient* method [6]–[8], and develop a trajectory simplification method based on the learned policy. We call this method *RLTS*. Compared with existing algorithms, our RLTS method computes simplified trajectories with smaller errors, which is mainly due to its data-driven nature and also its capability of adapting to different dynamics of the inputted points. The RLTS method has the time complexity of $O((n-W)\log W)$, where n is the number of points in the inputted trajectory and W is the buffer size, which is the same as that of existing algorithms STTrace, SQUISH, and SQUISH-E.

We also propose a variant of RLTS, called RLTS-Skip, by augmenting the MDP with additional actions of *skipping* points from being scanned. The rationale is that some points may carry little information so that they can be dropped immediately without being inserted in the buffer when they are being scanned, as RLTS does. The benefit is that the efforts of deciding and taking actions for these points that are skipped are saved and the efficiency is boosted. In addition, RLTS-Skip accepts a parameter J, which controls the maximum number of points that are allowed to be skipped. Therefore, RLTS-Skip provides a tunable trade-off between efficiency and effectiveness with different settings of J. Note that when $J = 0$, RLTS-Skip reduces to RLTS.

We further investigate the problem under the batch mode, which assumes more data access than under the online mode. Specifically, we have access to all points of a trajectory during the whole course of trajectory simplification in the batch mode, while we can only access those points that are stored in the buffer in the online mode. With the increased data access, we have more options of defining the states of the MDP. We investigate three state definitions, which capture different portions of the points of a trajectory, and correspondingly develop three different categories of algorithms, namely (1) RLTS and RLTS-Skip, (2) RLTS+ and RLTS-Skip+, and (3) RLTS++ and RLTS-Skip++. RLTS and RLTS-Skip are exactly the same as those for the online mode, whose underlying states are defined based on those points stored in the buffer only. RLTS+ and RLTS-Skip+ are enhanced versions of RLTS and RLTS-Skip, respectively, where their states are defined based on all those points that have been scanned (including those that are stored in the buffer and those that have been dropped). In addition, RLTS-Skip+ augments the states further with the information of the J points that follow the point that is being scanned. RLTS++ and RLTS-Skip++ ultimately consider all points for defining states. From RLTS and RLTS-Skip, to RLTS+ and RLTS-Skip+, to RLTS++ and RLTS-Skip++, more information is captured for defining the states and correspondingly, the MDPs become more complex and the methods take more time costs. Specifically, the time complexities of the three categories of methods are $O((n-W)\log W)$, $O((n-W)(n'+\log W))$, $O((n-W)(n'+\log n))$, respectively, where n' is the cost of computing the value of a point and is bounded by n (in practice, $n' \leq n$).

In summary, our main contribution is as follows. We develop a RL-based method for the trajectory simplification problem, which is the first of its kind. The RL-based method achieves competitive effectiveness and efficiency simultaneously. In particular, for the batch mode, the RL-based method outperforms *all* existing approximate methods in terms *both* effectiveness and efficiency, under *four* error measurements, across *different* parameter settings, and on *all* real datasets tested. Furthermore, we propose a variant of RLTS, i.e., RLTS-Skip, which runs faster than RLTS and provides a controllable trade-off between the effectiveness and efficiency. Particularly for the batch mode, we investigate three variants of the method given the increased data access. Extensive experiments on real-life datasets demonstrate that our methods have better effectiveness and comparable (or better in the batch mode) efficiency compared with existing algorithms.

The rest of paper is organized as follows. We review the literature in Section II and give the problem definition in Section III. We introduce our RLTS and RLTS-Skip algorithms for the online and batch modes in Section IV and in Section V, respectively. We present our experimental results in Section VI and finally conclude our paper in Section VII.

II. RELATED WORK

A. Trajectory Simplification in Online Mode

In the online mode, streaming trajectory data is continuously collected by sensors and stored in a local buffer. The trajectory simplification problem is to decide which points to be dropped and correspondingly which to be kept in the buffer and sent to the server's side later on. Among the existing studies of trajectory simplification in the online mode, [3]–[5] target the Min-Error problem. In [3], [9], the STTrace algorithm is proposed, which processes incoming points one by one and for each one, it first decides whether to drop it. If so, it moves to the next one; and if not, it drops one existing point in the buffer, and then inserts the point that is being processed to the buffer. The decision making is based on some heuristic values defined for the points. In [4], the authors propose the SQUISH algorithm and in [5] they propose an enhanced version of SQUISH called SQUISH-E. SQUISH and SQUISH-E follow the framework of STTrace, but use different definitions for the heuristic values of points. Each of these algorithms has the time complexity of $O((n-W)\log W)$. Our solution differs from these methods in that it is based on a policy learned via reinforcement learning instead of human-crafted heuristic values for decision making.

Other proposals on the online mode trajectory simplification do not target the Min-Error problem and are reviewed as follows. In [10]–[17], the authors address the problem of finding a simplified trajectory such that the size is minimized while satisfying a given error bound, i.e., a dual problem of the Min-Error problem. In [18], the "dead reckoning" technique is used for trajectory simplification, which predicts the future location of a moving object based on the assumption of a constant velocity and direction and discards a collected point if it deviates from the predicated location significantly. In [19], the authors propose to perform trajectory simplification based

on some topologically persistent features, which indicate the importance of points.

B. Trajectory Simplification in Batch Mode

In the batch mode, all the points in the trajectory that is to be simplified are inputted together and remain accessible throughout the simplification process. Among those existing studies of trajectory simplification in the batch mode, [10], [20]–[22] study the Min-Error problem. Specifically, in [20], the authors propose a dynamic programming algorithm called *Bellman*. Bellman runs in at least cubic time, which is prohibitively expensive for large datasets. In [9], [10], [21], the authors explore different approximate algorithms for the problem, including *Top-Down* and *Bottom-Up*. Top-Down is inspired by the traditional *Douglas-Peucker* [23] algorithm and the idea is to start with two points (the first one and the last one) and then repetitively include a point that has the largest error until the number of points reaches the storage budget. Top-Down has the time complexity of $O(Wn)$. Bottom-Up starts with all points of the inputted trajectory and repetitively drops a point that would introduce the smallest error until the number of points left is within the storage budget. Bottom-Up has the time complexity of $O((n-W)(n'+\log n))$, where n' is bounded by n. In [22], the authors propose an approximate algorithm called *Span-Search*, which is specifically designed for the error measurement direction-aware distance (DAD). Span-Search has the time complexity of $O(cn\log^2 n)$, where c is a moderate constant. Again, these methods are mainly based on human-crafted rules, while our method is based on a learned policy.

Other trajectory simplification methods for the batch mode do not target the Min-Error problem and are reviewed as follows. In [24], [25] (and the references therein), the authors study the dual problem of Min-Error. In [26], the authors propose a solution for selecting a subset of most representative points from a trajectory. In [27], the authors construct a reference trajectory set to support trajectory compression.

C. Road Network based Trajectory Compression

Trajectory compression [28]–[33] is a related but different problem. It aims to match the GPS points to road segments and leverage the knowledge from road networks to achieve a higher compression ratio. For example, Li et al [28] study uncertain trajectory compression and develop a framework to support probabilistic query processing in road networks. Chen et al [29] propose an online algorithm for trajectory mapping and compression utilizing vehicle heading directions upon the underlying road network. This line of research focuses on trajectory data that is generated on road networks while our work focuses movements in a free space, such as those of pedestrians, sports players, animals, etc., which are common and have been extensively studied.

D. Reinforcement Learning

Reinforcement learning (RL) was proposed to guide agents on what actions to take in a specific environment to maximize a cumulative reward [34], where the environment is generally modeled as a Markov decision process (MDP) [35]. In recent years, RL models have been used successfully to solve algorithm problems. For example, Kong et al. [36] explore the RL methods for three classic combinatorial optimization problems. Wang et al. [37] design RL-based algorithms for subtrajectory similarity search. Marcus et al. [38] apply RL for join order enumeration. In this paper, we model the trajectory simplification problem as an MDP and use a popular policy gradient method [6]–[8] for solving the problem. To the best of our knowledge, this is the first deep reinforcement learning based solution for trajectory simplification.

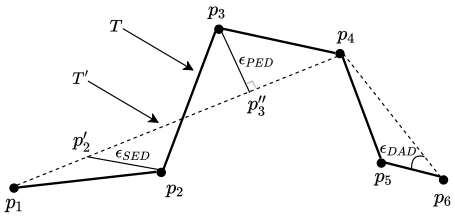


Fig. 1. A running example.

III. PRELIMINARIES AND PROBLEM STATEMENT

The trace of a moving object such as a vehicle or a user is usually captured by a trajectory, which corresponds to a sequence of time-stamped locations called *spatio-temporal points* (or simply *points*). Let $T =< p_1, p_2, ..., p_n >$ be a trajectory, where p_i is in the form of (x_i, y_i, t_i), meaning that a moving object is at location (x_i, y_i) at time t_i. We denote by $T[i:j]$ $(i \leq j)$ the subtrajectory of T, which starts form point p_i and ends at point p_j, i.e., $T[i:j] =< p_i, p_{i+1}, ..., p_j >$. We denote by $d(p_i, p_j)$ $(1 \leq i, j \leq n)$ the Euclidean distance between p_i's location and p_j's location. We define the size of the trajectory T, denoted by $|T|$, as the number of points involved in T, i.e., $|T| = n$. We denote the line segment linking location (x_i, y_i) and location (x_{i+1}, y_{i+1}) by $\overline{p_i p_{i+1}}$. It is interpreted that during the time period $[t_i, t_{i+1}]$ $(1 \leq i \leq n-1)$, the object moves along the line segment $\overline{p_i p_{i+1}}$ at a constant speed from one end to the other and the speed is equal to $\frac{d(p_i, p_{i+1})}{t_{i+1}-t_i}$.

A. Trajectory Simplification and Error Measurements

Any trajectory resulted from T by dropping some points (that are neither the first nor the last point) corresponds to a *simplified trajectory* of T. A simplified trajectory of T, denoted by T', has the form of $< p_{s_1}, p_{s_2}, ..., p_{s_m} >$ where $m \leq n$ and $1 = s_1 < s_2 < ... < s_m = n$. For example, in Figure 1, $T =< p_1, p_2, ..., p_6 >$ is a trajectory and $T' =< p_1, p_4, p_6 >$ is a simplified trajectory of T.

Consider the time period $[t_{s_j}, t_{s_{j+1}}]$ $(1 \leq j \leq m-1)$. Based on the simplified trajectory T', it is interpreted that the object moves along segment $\overline{p_{s_j} p_{s_{j+1}}}$, while based on trajectory T, it is interpreted that the object moves along a sequence of segments formed by a sequence of points $p_{s_j}, p_{s_j+1}, ..., p_{s_{j+1}}$. In other words, segment $\overline{p_{s_j} p_{s_{j+1}}}$ in T' *approximates* those segments starting at points $p_{s_j}, p_{s_j+1}, ..., p_{s_{j+1}-1}$, namely

$\overline{p_{s_j}p_{s_j+1}}, \overline{p_{s_j+1}p_{s_j+2}}, ..., \overline{p_{s_{j+1}-1}p_{s_{j+1}}}$, in T. We say that segment $\overline{p_{s_j}p_{s_{j+1}}}$ is an *anchor segment* of each of the points $p_{s_j}, p_{s_j+1}, ..., p_{s_{j+1}-1}$. Note each point in trajectory T except for p_n has exactly one anchor segment in T'. For example, in Figure 1, $\overline{p_1p_4}$ approximates a sequence of three segments $\overline{p_1p_2}$, $\overline{p_2p_3}$, $\overline{p_3p_4}$ and corresponds to the anchor segment of p_1, p_2 and p_3. As could be noticed, there is a discrepancy between the interpreted movement based on the simplified trajectory T' and that based on the original trajectory T, and this discrepancy is measured as the *error* of T' wrt T, which we denote by $\epsilon(T')$.

Quite a few measurements have been proposed for defining $\epsilon(T')$, including (1) synchronized Euclidean distance (SED) [3]–[5], [10], (2) perpendicular Euclidean distance (PED) [10]–[12], [20], (3) direction-aware distance (DAD) [13], [14], [22], [25], and (4) speed-aware distance (SAD) [5]. These error measurements share the idea that (1) it defines the error of a segment $\overline{p_{s_j}p_{s_{j+1}}}$ wrt a point p_i ($s_j \leq i < s_{j+1}$) which takes $\overline{p_{s_j}p_{s_{j+1}}}$ as its anchor segment, which we denote by $\epsilon(\overline{p_{s_j}p_{s_{j+1}}}|p_i)$; (2) it defines the error of the segment $\overline{p_{s_j}p_{s_{j+1}}}$, which we denote by $\epsilon(\overline{p_{s_j}p_{s_{j+1}}})$, as the maximum among its errors wrt the points that take it as an anchor segment, i.e., $\epsilon(\overline{p_{s_j}p_{s_{j+1}}}) = \max_{s_j \leq i < s_{j+1}} \epsilon(\overline{p_{s_j}p_{s_{j+1}}}|p_i)$; (3) it then defines the error of simplified trajectory T' as the maximum error of a segment in T', i.e., $\epsilon(T') = \max_{1 \leq j \leq m-1} \epsilon(\overline{p_{s_j}p_{s_{j+1}}})$. That is, these error measurements, including SED, PED, DAD, and SAD, define $\epsilon(T')$ as follows.

$$\epsilon(T') = \max_{1 \leq j \leq m-1} \max_{s_j \leq i < s_{j+1}} \epsilon(\overline{p_{s_j}p_{s_{j+1}}}|p_i) \quad (1)$$

Different measurements use different functions for defining $\epsilon(\overline{p_{s_j}p_{s_{j+1}}}|p_i)$ ($1 \leq j \leq m-1, s_j \leq i < s_{j+1}$).

- **SED** defines $\epsilon(\overline{p_{s_j}p_{s_{j+1}}}|p_i)$ as the Euclidean distance between p_i's location, i.e., (x_i, y_i), and the location at time t_i based on the movement interpreted by T', i.e., p_i's *synchronized* location based on T', which we denote by p'_i. In Figure 1, $\epsilon_{SED}(\overline{p_1p_4}|p_2)$ corresponds to $d(p_2, p'_2)$.
- **PED** defines $\epsilon(\overline{p_{s_j}p_{s_{j+1}}}|p_i)$ as the Euclidean distance between p_i's location, i.e., (x_i, y_i), and the location on segment $\overline{p_{s_j}p_{s_{j+1}}}$, that is the *closest* from (x_i, y_i), denoted by p''_i. In Figure 1, $\epsilon_{PED}(\overline{p_1p_4}|p_3)$ corresponds to $d(p_3, p''_3)$.
- **DAD** defines $\epsilon(\overline{p_{s_j}p_{s_{j+1}}}|p_i)$ as the *angular difference* between the direction along segment $\overline{p_ip_{i+1}}$ and that along segment $\overline{p_{s_j}p_{s_{j+1}}}$. In Figure 1, $\epsilon_{DAD}(\overline{p_4p_6}|p_5)$ corresponds to the angle between $\overline{p_4p_6}$ and $\overline{p_5p_6}$.
- **SAD** defines $\epsilon(\overline{p_{s_j}p_{s_{j+1}}}|p_i)$ as the difference between the speed of segment $\overline{p_ip_{i+1}}$ and that of segment $\overline{p_{s_j}p_{s_{j+1}}}$. In Figure 1, $\epsilon_{SAD}(\overline{p_4p_6}|p_5)$ corresponds to $|\frac{d(p_4,p_6)}{t_6-t_4} - \frac{d(p_5,p_6)}{t_6-t_5}|$.

B. Problem Definition

We study the Min-Error problem as defined below.

Problem 1 (Min-Error): Given a trajectory $T =< p_1, p_2, ..., p_n >$ and a storage budget W, which is an integer, the **Min-Error** problem is to find a simplified trajectory $T' =< p_{s_1}, p_{s_2}, ..., p_{s_m} >$ where $m \leq n$ and $1 = s_1 < s_2 < ... < s_m = n$ such that $|T'| \leq W$ and $\epsilon(T')$ is minimized, and $\epsilon(T')$ can be defined by any of those existing error measurements including SED, PED, DAD, and SAD.

The Min-Error problem has two modes, namely the online mode and the batch mode, for different application scenarios.

Online mode. In the online mode, the trajectory to be simplified is fed to the system point by point in an online fashion and those points that have been dropped during the trajectory simplification process will no longer be accessible. This mode is commonly used in applications such as remote sensing, where the sensors collect points from time to time and are constrained by storage budget, network bandwidth and energy.

Batch mode. In the batch mode, all the points in the trajectory to be simplified are fed together, and remain accessible during the simplification process. This mode is usually used at a server's side and the purpose is to reduce the storage cost (e.g., after the simplification, the original trajectory is discarded) and/or the query processing cost (e.g., it performs queries on the simplified trajectory data instead of the original data).

In this paper, we target both the online mode (Section IV) and the batch mode (Section V).

IV. Algorithms for Online Mode

In this section, by Min-Error, we mean the problem in the online mode unless specified otherwise. In the online mode, points are inputted one by one in an online fashion, while only a buffer with size W is available, i.e., at most W points can be retained throughout the trajectory simplification process. We adopt an existing strategy [3]–[5] that for the first W points, we store them in the buffer directly and for each of the remaining points, since the buffer is already full, we drop one point in the buffer to release some space and then store the new point in the buffer. Different from those existing strategies, which use some human-crafted heuristic values for deciding which point to drop when the buffer is full, we aim to achieve a more intelligent method for this decision-making task. Specifically, we treat the trajectory simplification problem as a *sequential decision making process* and model it as a *Markov decision process* (MDP) [35] (Section IV-A), use a policy gradient method [6]–[8] for learning an optimal policy for the MDP (Section IV-B), and then develop an algorithm called *RLTS*, which uses the learned policy for the Min-Error problem (Section IV-C). In Section IV-D, we present a variant of RLTS, called *RLTS-Skip*, which boosts the efficiency of RLTS via skipping some points from being scanned.

A. Min-Error Modeled as an MDP

We model the Min-Error problem as an MDP, which consists of four components, namely *states*, *actions*, *transitions*, and *rewards* as defined below.

1) States: Consider a situation where there are W points $p_{s_1}, p_{s_2}, ..., p_{s_W}$ in the buffer and a newly inputted point p_i ($i > W$) is to be inserted into the buffer next. The task is to drop one point from the buffer and then insert the point p_i into

the buffer. Conceptually, it is equivalent to the process that we first append the point p_i to the buffer, i.e., the buffer becomes $p_{s_1}, p_{s_2}, ..., p_{s_W}, p_{s_{W+1}}$, where $p_{s_{W+1}} = p_i$, and then we drop one point p_{s_j} $(2 \leq j \leq W)$ from the buffer. Note that by the definition of trajectory simplification, we are not allowed to drop point p_{s_1}, i.e., p_1. An intuitive idea is to drop one of those points such that the error that is introduced as a consequence of the dropping operation is small. If we drop the point p_{s_j} $(2 \leq j \leq W)$, two existing segments $\overline{p_{s_{j-1}} p_{s_j}}$ and $\overline{p_{s_j} p_{s_{j+1}}}$ would be destroyed, one new segment $\overline{p_{s_{j-1}} p_{s_{j+1}}}$ would be created, and other segments are unchanged. Since the error of a simplified trajectory is determined by those of its segments and the error of a segment is further determined by its errors wrt the points in the original trajectory that take the segment as their anchor segments, the error of the newly created segment $\overline{p_{s_{j-1}} p_{s_{j+1}}}$ wrt the point p_{s_j}, i.e., $\epsilon(\overline{p_{s_{j-1}} p_{s_{j+1}}} | p_{s_j})$, captures the consequence of dropping p_{s_j} well.

Motivated by this, we define for each point p_{s_j} $(2 \leq j \leq W)$, a *value*, denoted by $v(p_{s_j})$, as follows.

$$v(p_{s_j}) := \epsilon(\overline{p_{s_{j-1}} p_{s_{j+1}}} | p_{s_j}) \tag{2}$$

We note that for DAD and SAD, $\epsilon(\overline{p_{s_{j-1}} p_{s_{j+1}}} | p_{s_j})$ depends on segment $\overline{p_{s_j} p_{s_j+1}}$. In the online mode, p_{s_j+1} may not be accessible, and thus we use segment $\overline{p_{s_j} p_{s_{j+1}}}$ instead (i.e., we measure the angular and speed difference between segment $\overline{p_{s_{j-1}} p_{s_{j+1}}}$ and segment $\overline{p_{s_j} p_{s_{j+1}}}$). A lower value means that once the point is dropped, the introduced error tends to be smaller and thus it should be dropped with a higher chance. Then, we define the state of the situation, which we denote by s, based on the values of the points in the buffer. An immediate idea is to incorporate the values of all $(W-1)$ points p_{s_j} $(2 \leq j \leq W)$ in the buffer for defining the state. However, this definition has two issues. First, since W is an input to the problem and for different problem instances, W is usually different. With this definition, the model that is defined for one input W would not be usable for other inputs different from W. Second, W is typically a moderate to large integer, e.g., it could be in thousands. With this definition, the state space would be huge and the model be hard to train.

We propose to define the state s as the set containing k lowest values, where k ($k \leq W-1$) is hyper-parameter that could be tuned, instead of the set containing all $(W-1)$ values. Specifically, we let π denote the permutation of $s_2, ..., s_W$ such that $v(p_{\pi(1)}), v(p_{\pi(2)}), ..., v(p_{\pi(W-1)})$ is a list of the values in an ascending order. Then, we define state s as follows.

$$s := \{v(p_{\pi(1)}), v(p_{\pi(2)}), ..., v(p_{\pi(k)})\} \tag{3}$$

With this definition, a state is of a fixed size that is independent from the problem input. In addition, the state space is controllable via the parameter k.

2) Actions: Suppose that there are $W+1$ points $p_{s_1}, p_{s_2}, ..., p_{s_{W+1}}$ in the buffer (conceptually) and $s = \{v(p_{\pi(1)}), v(p_{\pi(2)}), ..., v(p_{\pi(k)})\}$ is the corresponding state. Essentially, the task is to pick a point among $p_{s_2}, ..., p_{s_W}$ and drop it. An immediate idea is to define $(W-1)$ actions, each for a point p_{s_j} $(2 \leq j \leq W)$, but then there would be two issues similar to those when we discuss a straightforward method for defining a state, namely (1) the definition would be W-dependent and thus it is not flexible and (2) the action space would be large and thus the model is hard to train. In fact, it is intuitive to restrict our attention to those points with small values since dropping one of these points incurs a small consequent error. Therefore, we focus on those points with their values maintained in the state, i.e., $p_{\pi(1)}, p_{\pi(2)}, ..., p_{\pi(k)}$.

With all these, we define an action space containing k actions, each meaning to drop a point $p_{\pi(j)}$ $(1 \leq j \leq k)$. Formally, we define an action, which we denote by a, as follows.

$$a := j \quad (1 \leq j \leq k) \tag{4}$$

where the action $a = j$ means that it drops the point $p_{\pi(j)}$.

3) Transitions: Suppose a is an action to drop point p_{s_j} $(2 \leq j \leq W)$. After the action a is taken, W points are left in the buffer. When a new point p_i, is inserted, we need to compute a new state, which we denote by s', i.e., state s' would be the next state when action a is taken at state s. We update the state s to state s' as follows. Recall that a state is mainly about the values of the points in the buffer, and in order to compute the state s', we examine how the points in the buffer and their corresponding values would have changed after p_{s_j} is dropped and a new point p_i is inputted.

First, we consider the consequence of dropping p_{s_j}. After the point p_{s_j} is dropped, only two neighboring points, namely $p_{s_{j-1}}$ and $p_{s_{j+1}}$, could have their anchor segment changed. Specifically, point $p_{s_{j-1}}$'s (if $j-1 \geq 2$) anchor segment would be changed from $\overline{p_{s_{j-2}} p_{s_j}}$ to $\overline{p_{s_{j-2}} p_{s_{j+1}}}$ and point $p_{s_{j+1}}$'s (if $j+1 \leq W-1$) anchor segment would be changed from $\overline{p_{s_j} p_{s_{j+2}}}$ to $\overline{p_{s_{j-1}} p_{s_{j+2}}}$. Therefore, the values of the two points need to be updated, which we do as follows.

$$v(p_{s_{j-1}}) = \max\{\epsilon(\overline{p_{s_{j-2}} p_{s_{j+1}}} | p_{s_{j-1}}), \epsilon(\overline{p_{s_{j-2}} p_{s_{j+1}}} | p_{s_j})\} \tag{5}$$

$$v(p_{s_{j+1}}) = \max\{\epsilon(\overline{p_{s_{j-1}} p_{s_{j+2}}} | p_{s_{j+1}}), \epsilon(\overline{p_{s_{j-1}} p_{s_{j+2}}} | p_{s_j})\} \tag{6}$$

Here, we include the two errors (i.e., $\epsilon(\overline{p_{s_{j-2}} p_{s_{j+1}}} | p_{s_j})$ and $\epsilon(\overline{p_{s_{j-1}} p_{s_{j+2}}} | p_{s_j})$) wrt the point p_{s_j} for updates. The rationale is as follows. Recall that $v(p_{s_{j-1}})$ is defined to capture the consequence of dropping $p_{s_{j-1}}$. Ideally, $v(p_{s_{j-1}})$ should be defined as the error of the segment $\overline{p_{s_{j-2}} p_{s_{j+1}}}$, i.e., $\max_p \epsilon(\overline{p_{s_{j-2}} p_{s_{j+1}}} | p)$, where p is a point that takes segment $\overline{p_{s_{j-2}} p_{s_{j+1}}}$ as the anchor segment. In the online mode, among those points that take segment $\overline{p_{s_{j-2}} p_{s_{j+1}}}$ as the anchor segment, only $p_{s_{j-1}}$ and p_{s_j} are accessible when p_{s_j} is being dropped. Therefore, we include $\epsilon(\overline{p_{s_{j-2}} p_{s_{j+1}}} | p_{s_j})$ for defining $v(p_{s_{j-1}})$. Similarly, we include $\epsilon(\overline{p_{s_{j-1}} p_{s_{j+2}}} | p_{s_j})$ for defining $v(p_{s_{j+1}})$.

Second, we consider the consequence of inserting point p_i. With point p_i inserted to the buffer, there would be $W+1$ points, which we still denote by $p_{s_1}, p_{s_2}, ..., p_{s_{W+1}}$. Note that $p_{s_{W+1}}$ corresponds to p_i. The value of p_{s_W}, which is previously not defined (since it is the last point in the buffer before p_i is inserted), needs to be defined as follows.

$$v(p_{s_W}) = \epsilon(\overline{p_{s_{W-1}} p_{s_{W+1}}} | p_{s_W}) \tag{7}$$

The values of all other points are unchanged. Based on these values, we compute the state s' in the same way as we compute the state s (Section IV-A1).

4) Rewards: Consider that we perform an action a at a state s and then we arrive at a new state s'. We define the reward associated with this transition from state s to state s', which we denote by r, as follows. At state s, we have W points in the buffer and a new point p_i to be inserted. We consider those points in the buffer, which constitute a trajectory that corresponds to a simplified trajectory of the trajectory fed so far, i.e., $T[1:i-1]$. We denote this simplified trajectory by T'. Similarly, at state s', we have a simplified trajectory of $T[1:i]$, which we denote by T''. We then define the reward r as follows.

$$r = \epsilon(T') - \epsilon(T'') \tag{8}$$

where $\epsilon(T')$ is wrt $T[1:i-1]$ and $\epsilon(T'')$ is wrt $T[1:i]$. The intuition is that if the error of the simplified trajectory resulted from the action, i.e., $\epsilon(T'')$, is smaller, then the reward is larger. With this definition, it would favor those actions that lead to simplified trajectories with smaller errors. In fact, it could be verified that with this reward definition, the goal of the MDP problem is well aligned with that of the trajectory simplification problem. To see this, suppose that we go through a sequence of states $s_1, s_2, ..., s_N$ and correspondingly, we receive a sequence of rewards $r_1, r_2, ..., r_{N-1}$. In the case that the future rewards are not discounted, we have

$$\sum_{t=1}^{N-1} r_t = \sum_{t=1}^{N-1} (\epsilon(T'_t) - \epsilon(T''_t)) = \epsilon(T'_1) - \epsilon(T''_{N-1}) = -\epsilon(T''_{N-1}) \tag{9}$$

where T'_t (resp. T''_t) is the simplified trajectory at the state s_t before (resp. after) the action a_t is performed. Note that $\epsilon(T'_1) = 0$ since at the start state, no points have been dropped and thus the error is equal to 0, and $\epsilon(T''_{N-1})$ corresponds to the error of the simplified trajectory of T.

Remarks. We note that for the computation of the reward r, which involves the computations of the errors of two simplified trajectories, is only required for the learning process. Therefore, we can use a repository of trajectories for the learning process, and once a policy has been learned, we use it for trajectory simplification. In addition, we note that we can compute $\epsilon(T'_t)$'s and $\epsilon(T''_t)$'s *incrementally* except for $\epsilon(T'_1)$ since T''_t corresponds to T'_t with one point dropped ($1 \leq t \leq N$) and T'_{t+1} corresponds to T''_t with one point inserted ($1 \leq t \leq N-1$).

B. Policy Learning on the MDP

The core problem of an MDP is to find an optimal *policy* for the agent, which corresponds to a function that specifies the action that the agent should choose at a specific state so as to maximize the accumulative rewards. We learn the policy for the MDP via a *policy gradient* (PNet) method, which is widely used [6]–[8]. PNet models a stochastic policy as $\pi_\theta(a|s)$, which means the probability of selecting an action a for a given state s. PNet parameterizes $\pi_\theta(a|s)$ using a neural network as follows.

$$\pi_\theta(a|s) = \sigma(\boldsymbol{W}s + \boldsymbol{b}) \tag{10}$$

where σ denotes the softmax function and $\theta = \{\boldsymbol{W}, \boldsymbol{b}\}$ denotes the parameters of the neural network. Then, PNet computes the gradients of some performance measure wrt the parameters θ as follows.

$$\nabla_\theta J(\theta) = \sum_{t=1}^{N} \frac{R_t - \overline{R}}{\sigma_R} \nabla_\theta \ln \pi_\theta(a_t|s_t) \tag{11}$$

where $s_1, s_2, ..., s_N$ is a sequence of states, $a_1, a_2, ..., a_{N-1}$ is a sequence of actions by sampling $\pi_\theta(a|s_t)$, R_t denotes the accumulative reward since action a_t is taken, $\overline{R}$ is the mean of R_t's, and σ_R is the standard deviation of R_t's. PNet repeatedly updates the parameters θ via *gradient ascent* based on the gradients computed in Equation (11) until some stopping criterion specified by users is satisfied. Note that PNet corresponds to a variant of the REINFORCE algorithm with baseline [6]–[8] and the normalization mechanism based on the mean and standard deviation helps reduce the variance of the computed gradients in Equation (11).

C. The RLTS Algorithm

Our RLTS algorithm is based on the learned policy for the MDP that models the Min-Error problem, which is presented in Algorithm 1. Specifically, it stores the first W points in the buffer directly (Line 1 - 3). It initializes an index t for a sequence of states and actions to be traversed (Line 4); Then, for each of the following points, says, p_i, it proceeds as follows (Line 5). It computes (incrementally if possible) the values of the points in the buffer except for the first one, i.e., $v(p_{s_j})$ ($2 \leq j \leq W$), and maintains the values in a min-priority queue with the ascending permutation denoted by π (Line 6). It then constructs a state s_t containing the set of k lowest values of points (Line 7), and samples an action a_t based on the stochastic policy that has been learned (Line 8). Based on the sampled action $a_t = j$, it drops the point with the j^{th} lowest value, i.e., $p_{\pi(j)}$ (Line 9). It then inserts the point p_i that is being processed (Line 10). Finally, it returns a simplified trajectory T' which involves all the W points in the buffer (Line 13 - 14).

Time complexity. The time complexity of the RLTS algorithm is $O((n-W)\log W)$, where n is the number of points in the input trajectory T and W denotes the storage budget of the buffer. To see this, the complexity is dominated by the part of processing the last $(n-W)$ points (Line 5 - 12 in Algorithm 1), and the time cost of processing one point consists of (1) that of computing the state whose cost is $O(1)$ incrementally (Line 6); (2) that of maintaining the min-priority queue is $O(\log W)$ (Line 6); and (3) that of constructing a state, sampling an action, dropping a point, inserting a point and updating the index is $O(1)$ assuming k is a constant (Line 7 - 10). We note that this time complexity is the same as those of existing algorithms [3]–[5] and all algorithms can

Algorithm 1 The RLTS algorithm

Require: A trajectory $T =< p_1, p_2, ..., p_n >$ which is inputted in an online fashion; A buffer with a storage budget W ($W < n$);

Ensure: A simplified trajectory T' of T with $|T'| \leq W$;

1: **for** $i = 1, 2, ..., W$ **do**
2: Store point p_i into the buffer;
3: **end for**
4: $t \leftarrow 1$;
5: **for** $i = W + 1, W + 2, ..., n$ **do**
6: Compute (incrementally if possible) the values of the points in the buffer except for the first one, i.e., $v(p_{s_j})$ ($2 \leq j \leq W$) and maintain the values in a min-priority queue with the ascending permutation denoted by π;
7: Construct a state $s_t \leftarrow \{v(p_{\pi(1)}), v(p_{\pi(2)}), ..., v(p_{\pi(k)})\}$;
8: Sample an action $a_t \sim \pi_\theta(a|s)$;
9: Drop the point $p_{\pi(j)}$ from the buffer where $a_t = j$;
10: Insert the point p_i into the buffer;
11: $t \leftarrow t + 1$;
12: **end for**
13: Trajectory $T' \leftarrow$ the sequence of points in the buffer;
14: **Return** trajectory T';

meet practical requirements as shown in our experiments (e.g., the time of processing one point is much less than 1ms on a moderate machine). Compared with existing algorithms, RLTS is based on a learned policy but not some human-crafted rules, and thus it could return simplified trajectories with smaller errors.

To analyze the time cost of learning the policy, we first consider the time cost of learning on one trajectory of length n for one epoch. The time cost should be $O((n-W)(n+\log W))$ since compared with the RLTS algorithm, the training process involves two additional costs, namely that for computing the reward after an action is taken and that for performing the gradient decent over a neural network. The former has the cost of $O(n)$ and the latter has a constant cost given that the neural network we use has a small size that is independent of the problem size. Let N_t be the number of trajectories and E be the number of epochs for training the policy. The time cost of training is $O(E \cdot N_t \cdot (n - W)(n + \log W))$.

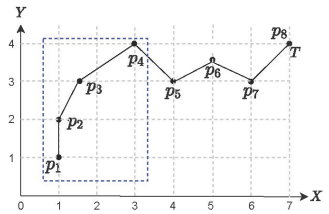
D. The RLTS-Skip Algorithm

In the RLTS algorithm, each point is inserted to the buffer for sure after it is scanned. It may then be dropped when some following points are being scanned. Consequently, when each point is being scanned, some efforts are spent on deciding which existing point in the buffer to be dropped by going through a neural network in RLTS. While this strategy gives each point a chance to be included in the buffer and thus exploring a large space of possible simplified trajectories, it may be too conservative. Consider a scenario where the points that are scanned most recently constitute a trajectory that indicates a movement along a straight line with a constant speed. In this scenario, we have much confidence to drop a certain number of points in a row without including them one by one to the buffer and then dropping some of them at later stages. This would help save the efforts for deciding and taking actions when scanning these points and at the same time, the effectiveness should not be affected much.

Motivated by this, we propose to augment the MDP that is defined in Section IV-A by introducing J additional actions when scanning a point p_i, namely (1) dropping p_i and continuing to scan p_{i+1}, (2) dropping p_i and p_{i+1} and continuing to scan p_{i+2}, ..., and (J) dropping p_i, p_{i+1}, ..., and p_{i+J-1} and continuing to scan p_{i+J}. Here, J is a hyper-parameter, which could be tuned. These actions essentially mean (1) skipping 1 point, (2) skipping 2 points, ..., and (J) skipping J points during the process of scanning the points of a trajectory sequentially. As a result, the augmented MDP involves $(k+J)$ actions, namely the k actions of the original MDP as defined in Section IV-A2 and the J actions as newly introduced in this section. Note that these $(k + J)$ actions are exclusive and at each state, only one of them could be taken. For an action of dropping a point, the reward is defined the same as in the online mode. For an action of skipping j points, the reward is still defined by Equation (8), but with T'' changed to be the points stored in the buffer plus p_{i+j}, which corresponds to a simplified trajectory of $T[1 : i+j]$.All other components of the original MDP remain unchanged. We call the reinforcement learning algorithm based on this augmented MDP as *RLTS-Skip*. We note that when J is set to 0, RLTS-Skip reduces to RLTS.

The RLTS-Skip algorithm is illustrated in Figure 2 with the inputted data trajectory shown at the left side and $W = 4$. It first stores 4 points p_1, p_2, p_3 and p_4 in the buffer. It then scans point p_5, observes the first state s_1 and takes the action of dropping the point p_3. It then scans point p_6, updates the values of two neighboring points of p_3, namely p_2 and p_4, and computes the value of the newly inserted point p_5. It then observes the second state s_2 and takes the action of skipping the next 2 points, i.e., p_6 and p_7. It then scans p_8 and updates the value of p_5. It observes the third state s_3, takes an action of dropping p_2, and then terminates. At the end, it returns a simplified trajectory $T' =< p_1, p_4, p_5, p_8 >$, which has the PED error 0.693.

As could be verified, RLTS-Skip has the same time complexity as RLTS. But in practice, RLTS-Skip should have better efficiency than RLTS for two reasons: (1) At each state, RLTS-Skip can take either a "dropping" action (among the k actions) or a "skipping" action (among the J actions), RLTS can only take a "dropping" action, and the cost incurred by a "skipping" action is smaller than that by a "dropping" action (since for a "dropping" action, the values of three points need to be updated (Equation (5), (6), and (7)) while for a "skipping" action, only the value of one point need to be updated (Equation (7)); and (2) For RLTS-Skip, for those points that are skipped, the efforts for deciding and taking an action are saved.



Initial	Store $< p_1, p_2, p_3, p_4 >$ into the buffer with the initial reward 0.0				
Point	State	Action	Buffer	Reward	Accum Reward
p_5	$s_1 = \{v(p_2) = 0.243, v(p_3) = 0.354, v(p_4) = 1.0\}$	Drop p_3	$< p_1, p_2, p_4, p_5 >$	-0.354	-0.354
p_6	$s_2 = \{v(p_2) = 0.693, v(p_5) = 0.728, v(p_4) = 1.265\}$	Skip p_6 and p_7	$< p_1, p_2, p_4, p_5 >$	-0.278	-0.632
p_7	-	-	-	-	-
p_8	$s_3 = \{v(p_2) = 0.693, v(p_5) = 1.0, v(p_4) = 1.265\}$	Drop p_2	$< p_1, p_4, p_5, p_8 >$	-0.061	-0.693
Output	Return $T' =< p_1, p_4, p_5, p_8 >$ with $\epsilon(T') = 0.693$				

Fig. 2. Illustration of the RLTS-Skip algorithm with PED.

V. Algorithms for Batch mode

In the batch mode, we have more data access than in the online mode. Specifically, we have access to all points of a trajectory during the course of trajectory simplification in the batch mode, while in the online mode, we can only access those points that are stored in the buffer. With the increased data access, we have more options of defining the states of the MDP. In the following, we investigate three state definitions, which capture different portions of the points of a trajectory, and correspondingly develop three different sets of algorithms, namely (1) RLTS and RLTS-Skip, (2) RLTS+ and RLTS-Skip+, and (3) RLTS++ and RLTS-Skip++.

(1) RLTS and RLTS-Skip. The RLTS and RLTS-Skip algorithms that are designed for the online mode can immediately carry over for the batch mode. Same as the case of online mode, the states of the MDP underlying RLTS and RLTS-Skip are defined based on those points that are stored in the buffer only. The time complexity of RLTS and RLTS-Skip is $O((n-W)\log W)$.

(2) RLTS+ and RLTS-Skip+. In RLTS, a state is defined based on the values of the points in the buffer and the value of a point is defined as the error of its anchor segment wrt the point only. That is, the errors of the point's anchor segment wrt other points that take the segment as an anchor segment are ignored since those points have been dropped already and are no longer accessible in the online mode. In the batch mode, this is not the case. In fact, we can make use of these errors for defining the value of a point so as to capture richer information. Specifically, we define the value of a point p_{s_j} ($2 \leq j \leq W$) in the buffer as the maximum error of the p_{s_j}'s anchor segment wrt a point that takes the segment as its anchor segment as follows.

$$v(p_{s_j}) := \max_{s_{j-1} \leq i < s_{j+1}} \epsilon(\overline{p_{s_{j-1}} p_{s_{j+1}}} | p_i) \tag{12}$$

With this new definition of the value of a point, we would have correspondingly a new MDP, a new learned policy and a new algorithm for trajectory simplification. We call this algorithm *RLTS+*. Essentially, RLTS+ is an adapted version of RLTS with the states enhanced with additional information of those points that have been dropped before. The time complexity of RLTS+ is $O((n-W)(n' + \log W))$, where n' is the cost of computing the value of a point and is bounded by n (in practice, $n' \leq n$).

Similarly, we refine the state definition of the MDP for RLTS-Skip by (1) using the new definition of the value of a point (Equation (12)); and (2) appending J values to the original k values, each corresponding to the error incurred by dropping j points $p_i, ..., p_{i+j-1}$ for $1 \leq j \leq J$, when scanning point p_i. We then develop an algorithm based on the MDP with the refined state. We call the resulting algorithm *RLTS-Skip+*. The time complexity of RLTS-Skip+ is the same as RLTS+.

(3) RLTS++ and RLTS-Skip++. In both RLTS and RLTS+, a buffer of a *fixed* size (i.e., W) is maintained and only those points stored in the buffer are considered as candidates to drop. This is necessary in the online mode due to the restricted data access. In the batch mode, all points can be accessed throughout the course of trajectory simplification. Therefore, an alternative design is to use a buffer of a *variable* size. Specifically, we put all the points in the buffer at the beginning and each time we drop one point from the buffer until only W points remain in the buffer. Based on this design, we can define the states in the same way as we do for RLTS+ except that the buffer is of a variable size. Correspondingly, we can obtain a MDP, a learned policy and a trajectory simplification algorithm. We call the resulting algorithm *RLTS++*. Similarly, we can replace the buffer of RLTS-Skip+ with one of a variable size and to obtain a new algorithm, which we call *RLTS-Skip++*. For RLTS-Skip++, all points are stored in the buffer at the beginning, an action of skipping j points means dropping j points.

RLTS++ and RLTS-Skip++ come with an increased cost of computing a state since it needs to maintain the k lowest values among $O(n)$ values instead of W values as RLTS or RLTS+ does. As a result, the time complexity of RLTS++ and RLTS-Skip++ becomes $O((n-W)(n' + \log n))$, where n' is the cost of computing the value of a point and is bounded by n (in practice, $n' \leq n$).

Comparisons and Analysis. From RLTS, to RLTS+, to RLTS++, more information is captured for defining the states, and correspondingly, the resulting algorithms for trajectory simplification have more time costs, i.e., from $O((n-W)\log W)$, to $O((n-W)(n' + \log W))$, to $O((n-W)(n' + \log n))$. This holds for RLTS-Skip, RLTS-Skip+, and RLTS-Skip++ as well. As will be investigated in Section VI-A, these algorithms provide different trade-offs of effectiveness and efficiency. Among RLTS, RLTS+ and RLTS++, RLTS++ has the best effectiveness and RLTS has the best efficiency. Among RLTS-Skip, RLTS-Skip+ and RLTS-Skip++, RLTS-Skip++ has the best effectiveness and RLTS-Skip has the best efficiency. As can be verified, the time complexity of training for RLTS, RLTS+, RLTS-Skip, and RLTS-Skip+ is $O(E \cdot N_t \cdot (n-W)(n + \log W))$ and that for RLTS++ and RLTS-Skip++ is $O(E \cdot N_t \cdot (n-W)(n + \log n))$, where E is the number of epochs and N_t is the number of trajectories used for training.

TABLE I
DATASET STATISTICS.

Statistics	Geolife	T-Drive	Truck
# of trajectories	17,621	10,359	10,110
Total # of points	24,876,978	17,740,902	10,059,685
Ave. # of points per trajectory	1,412	1,713	995
Sampling rate	1s $\sim$ 5s	177s	3s $\sim$ 60s
Average distance	9.96m	623m	82.74m

VI. EXPERIMENTS

A. Experimental Setup

Dataset. Our experiments are conducted on three real-world trajectory datasets, namely Geolife, T-Drive and Trucks. Geolife[1] records the outdoor trajectories of 182 users for a period of five years. T-Drive [2] tracks the trajectories of 10,357 taxis in Beijing, and Truck [3] records the GPS trajectories of 10,368 trucks in China during a period from March to October, 2015. The three datasets are widely used in evaluating trajectory simplification [16], [22], [25] including the recent dedicated evaluation work [2] and the detailed statistics are summarized in Table I.

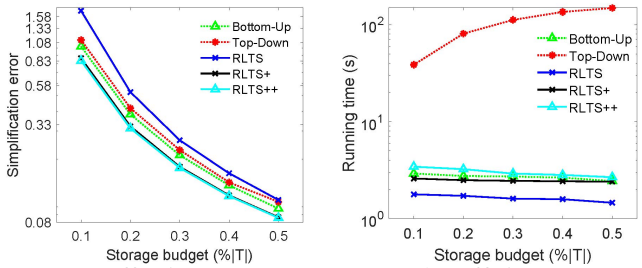
Algorithms for Comparison. We review the trajectory simplification literature thoroughly and identify seven methods for comparison, including (1) STTrace [3], (2) SQUISH [4] and (3) SQUISH-E [5] for the online mode and (4) Bellman [20], (5) Top-Down [39], (6) Bottom-Up [21], [25], and (7) Span-Search [22] for the batch mode. We cross check these algorithms against those covered in a recent survey [2] on trajectory simplification so that all existing methods that are proposed for the Min-Error problem are included for comparison. Note that we do not compare with the adaptions of the algorithms that are designed for the dual problem of Min-Error (via binary search) since these adapted algorithms would have the time complexity at least $O(n^2 \log n)$, e.g., for DAD, the time complexity is $O(n^2 C \log n)$, where $C < n$, which are clearly higher than those of RLTS and RLTS-Skip and not scalable on large datasets.

Parameter Setting and Policy Learning. The neural network used in the RLTS and RLTS-Skip (RLTS+ and RLTS-Skip+) methods involves one input layer, one hidden layer and one output layer, where the hidden layer involves 20 neurons and uses the tanh function as the activation function. In order to avoid the data scale issues, batch normalization provided by tensorflow is employed before the activation. For RLTS (RLTS+), the output layer involves k neurons and k is set as 3 by default. For RLTS-Skip (RLTS-Skip+), the output layer involves $(k + J)$ neurons and k and J are set as 3 and 2, respectively. We study the effects of these two hyper-parameters in Section VI-B. We randomly sample 1,000 trajectories from a training dataset, and for each trajectory, we generate 10 episodes for policy learning. Since each trajectory involves around 1,000 points, there would be about 10 million transition steps in the learning process. In addition, we use the Adam stochastic gradient descent with the learning rate of 0.001 based on empirical findings. For the reward discount factor, we tried several settings, and since the results are similar, we set it as 0.99. We take the policy, which gives the maximum reward per episode and use it for trajectory simplification. For the online mode, we sample an action with the probability outputted by the softmax function at each state, and for the batch mode, we take the action with the maximum probability based on empirical findings. The units of SED, PED, DAD, and SAD are 10m, 10m, 1 radian and 10m/s, respectively.

[1]http://research.microsoft.com/en-us/downloads/b16d359d-d164-469e-9fd4-daa38f2b2e13/

[2]http://research.microsoft.com/apps/pubs/?id=152883

[3]http://mashuai.buaa.edu.cn/traj.html



(a) Effectiveness (b) Efficiency

Fig. 3. Variants of RLTS (Batch mode)

Evaluation Platform. All the methods are implemented in Python 3.6. The implementation of RLTS and RLTS-Skip (RLTS+ and RLTS-Skip+) is based on tensorflow 1.8.0. The experiments are conducted on a machine with Intel(R) Xeon(R) CPU E5-1620 v2 @3.70GHz 16.0GB RAM and one Nvidia GeForce GTX 1070 GPU. The codes can be downloaded via the link https://github.com/zhengwang125/RLTS.

B. Experimental Results

(1) Comparison with the exact algorithm Bellman (Batch mode). Since Bellman has a cubic time complexity and is slow, we compare it with RLTS+ and RLTS-Skip+ on some small datasets only, for which we randomly select a set of 100 trajectories, each with around 300 points from Geolife. The results are put in the technical report [40] due to the page limit.We observe that the simplification error of RLTS+ and RLTS-Skip+ is very close to the exact algorithm Bellman for the four measurements; however, RLTS+ and RLTS-Skip+ run faster than Bellman by around three orders of magnitude. The results on the other datasets are qualitatively similar and thus omitted.

(2) Comparison among variants (batch mode). We randomly sample 1,000 trajectories from Geolife each with 5,000 points, run the algorithms, and report the average SED error and running time. The results are shown in Figure 3. We observe that the effectiveness increases and the efficiency drops from RLTS, to RLTS+, to RLTS++. This is because from RLTS, to RLTS+, to RLTS++, more information is captured for defining the states, and correspondingly the MDP is more powerful yet takes more time to run. Besides, only RLTS+ dominates the best existing algorithm, i.e., Bottom-UP, in terms of both effectiveness and efficiency. The results for

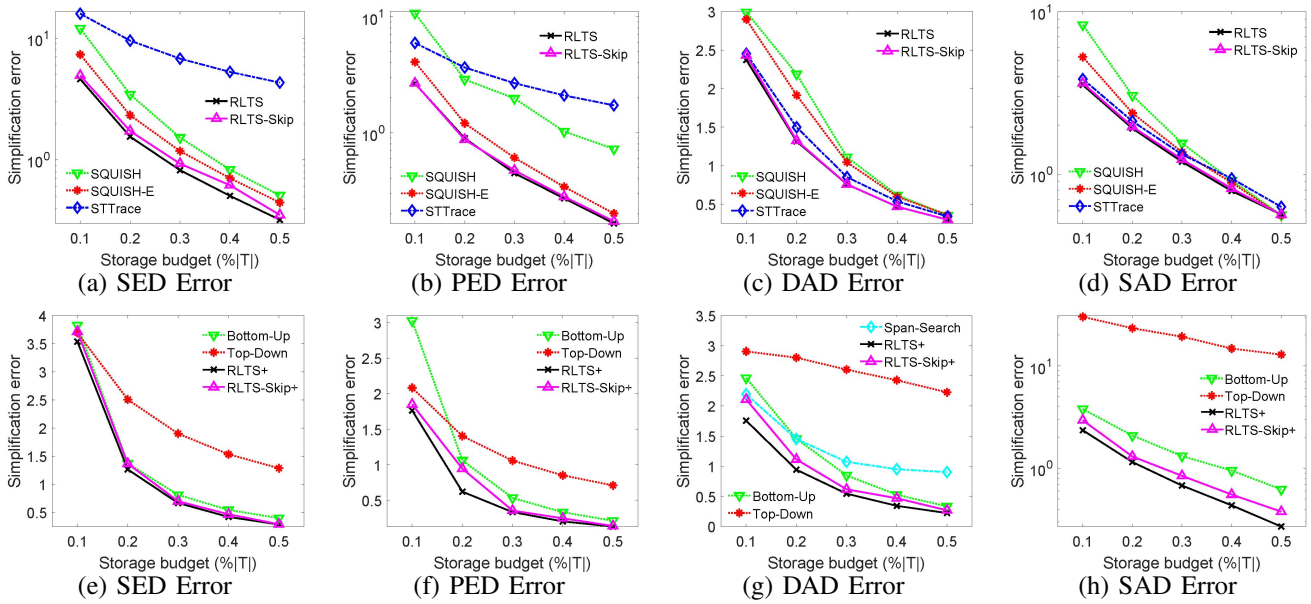


Fig. 4. Effectiveness evaluation with varying W ((a)-(d): Online mode, (e)-(h): Batch mode, Geolife).

RLTS-Skip, RLTS-Skip+, and RLTS-Skip++ show a similar trend and are put in the technical report [40] due to the page limit. Therefore, in the following experiments, we focus on RLTS+ and RLTS-Skip+ for the batch mode.

(3) Effectiveness evaluation (comparison with existing approximate algorithms). We randomly sample 1,000 trajectories T from a dataset and vary the storage budget W from $0.1 \times |T|$ to $0.5 \times |T|$ by following [22]. Figure 4 show the results for both online and batch modes on the Geolife dataset. The results on the other two datasets T-Drive and Truck demonstrate similar trends and are put in a technical report [40]. Overall, the results clearly show that RLTS (RLTS+) consistently outperforms existing algorithms under all error measurements for both online and batch modes and on all datasets. For RLTS-Skip (RLTS-Skip+), it beats all baselines for the online mode, and provides comparable performance in the batch mode. In addition, the effectiveness of RLTS-Skip (RLTS-Skip+) is slightly worse than RLTS (RLTS+), but still better than that of the baselines due to its data-driven nature.

(4-6) Effectiveness evaluation (the learned policy, parameter k and J). We conduct experiments to study the effect of the learned policy, parameter k, and parameter J on 1,000 randomly sampled trajectories from Geolife. The results and detailed description are included in the technical report [40] due to the page limit. We conclude from the results that: (1) the learned policy contributes significantly to the effectiveness especially in the online mode; (2) as k grows, the running time becomes larger and the accuracy becomes better as expected; (3) as J grows, RLTS-Skip has its effectiveness degrades and efficiency improves.

(7) Efficiency evaluation (varying the trajectory length $|T|$). We follow [2], [22] by varying $|T|$ from 10,000 to 50,000. For each setting, we randomly select 100 trajectories with the size around the setting from Truck. We fix W at $0.1|T|$.

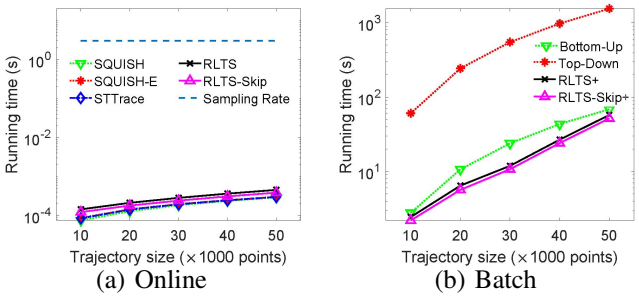


Fig. 5. Efficiency evaluation (varying $|T|$) on Truck.

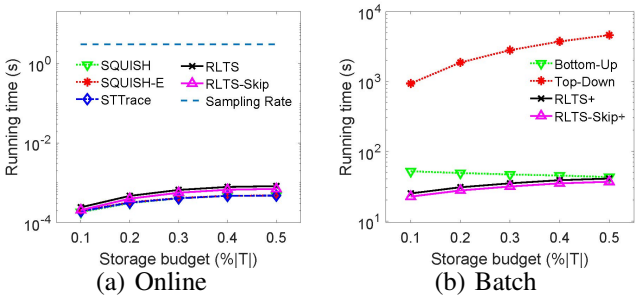
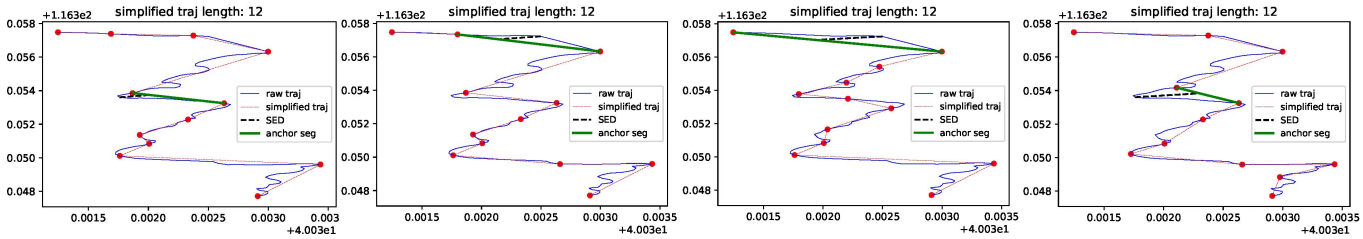


Fig. 6. Efficiency evaluation (varying W) on Truck.

Figure 5 shows the results for both online and batch modes under SED. In the online mode, we show the average running time per point because the processing time of a single point is important for an online scenario. In addition, the sampling rate (3s) of the Truck dataset is shown with a dotted line in the figure for reference. We observe that RLTS and RLTS-Skip are slightly slower than the three baseline algorithms though all of them have the same time complexity. This is because learning-based algorithms employ the learning models to make the decision (i.e., dropping or skipping) while the other three algorithms use a simple comparison operation for the same task. In addition, RLTS-Skip runs faster than RLTS since the time cost of constructing states and choosing actions are saved



(a) RLTS ($\epsilon = 2.851$) (b) RLTS-Skip ($\epsilon = 3.987$) (c) SQUISH, SQUISH-E ($\epsilon = 5.987$) (d) STTrace ($\epsilon = 5.866$)

Fig. 7. Case study (Online mode, Geolife).

TABLE II
TRAINING TIME ON GEOLIFE (HOURS).

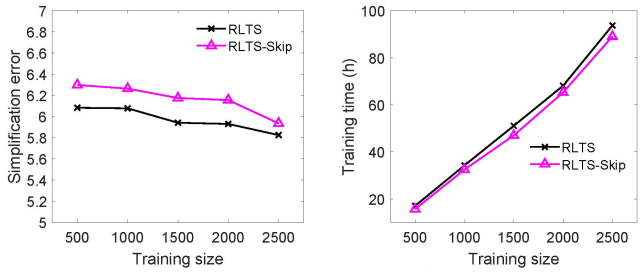
Measurement	SED		PED		DAD		SAD	
Mode	Online	Batch	Online	Batch	Online	Batch	Online	Batch
RLTS(+)	10.3	11.1	9.7	11.5	9.3	10.1	14.1	15.2
RLTS-Skip(+)	9.7	10.7	7.3	7.6	6.5	6.8	12.3	13.9

for those points that have been skipped. Overall, RLTS and RLTS-Skip are fast enough (very close to other algorithms) and far meets the practical needs, e.g., for a trajectory with about 10,000 points, they take less than 0.15ms per point, which is 20,000 times faster than the sampling rate (3s). In the batch mode, both RLTS+ and RLTS-Skip+ are faster than Top-Down and Bottom-Up, and the gaps of efficiency are aligned with the time complexities. We omit the running time of Span-Search because it has been shown to be slower than Top-Down in the existing studies [2], [22]. The results on the other datasets under other error measurements are qualitatively similar as those reported in Figure 5 and are omitted.

(8) Scalability test. We conduct the scalability test on long trajectories in the datasets. The results are included in the technical report [40] due to the page limit. Here, we present the running time results on the longest trajectory, which involves around 383,000 points, that RLTS-Skip+, RLTS+, Bottom-Up, and Top-Down run in 2,843s, 3,412s, 4,952s, 98,427s, respectively.

(9) Efficiency evaluation (varying the budget size W). We vary W from $0.1|T|$ to $0.5|T|$ on Truck and fix $|T|$ at 40,000 using SED as the error measurement. Figure 6 shows the result for both the online and batch modes. Similarly, in the online mode, RLTS and RLTS-Skip are a bit slower than SQUISH, SQUISH-E and STTrace, but run reasonably fast. For example, they take less than 1ms per point point for a trajectory with 40,000 points. The running times of all the methods slightly increase with W. In the batch mode, both RLTS+ and RLTS-Skip+ run faster than Top-Down by around two orders of magnitude. They also run faster than Bottom-Up, and the gap reduces as W increases since they take $O(\log W)$ time to construct states while Bottom-Up takes $O(\log n)$ time to decide which segments to merge.

(10) Case study. In Figure 7, blue solid lines indicate a raw trajectory and red dashed lines indicate its simplified trajectories by different algorithms in the online mode. We label the SED errors with black dashed lines. The results clearly show that our RL-based methods return better results than baselines. For example, the SED of RLTS ($\epsilon = 2.851$) is around half of those of SQUISH and SQUISH-E ($\epsilon = 5.987$),



(a) SED Error (b) Time cost

Fig. 8. Training cost on Geolife (Online mode).

and STTrace ($\epsilon = 5.866$). The results for the batch mode show similar clues and can be found in the technical report [40].

(11) Training time. The training times of RLTS and RLTS-Skip (RLTS+ and RLTS-Skip+) under different error measurements on Geolife are shown in Table II. It normally takes several hours to train a satisfactory RL model. The reported training times of RLTS-Skip are smaller than those of RLTS. This is because we use the same training samples and epochs for training both algorithms and RLTS-Skip runs faster.Further, we study how the number of training samples affects the model performance. We randomly sample 5 training sets from Geolife, which include 500, 1,000, 1,500, 2,000, 2,500 trajectories, respectively. For each training set, we report its training cost and the corresponding effectiveness with the default setup in Section VI-A. The results are shown in Figure 8. We observe that the effectiveness slightly improves with the number of training samples and the corresponding training cost increases almost linearly. We use the learned policy based on the 1,000 trajectories for other experiments because it gives a reasonable trade-off between effectiveness and training cost.

VII. CONCLUSION

In this paper, we study the trajectory simplification problem in both online and batch modes. We propose a reinforcement learning (RL)-based method called RLTS for both modes. Compared with existing algorithms, which are mainly heuristic-based, our RLTS method is data-driven and can adapt to different dynamics of the underlying points. We conduct extensive experiments, which show that RLTS computes simplified trajectories with consistently lower errors and runs comparably fast in the online mode and faster in the batch mode, compared with existing algorithms. One interesting direction for future research is to explore how to choose the error measurement (e.g., SED, PED, etc.) adaptively for different application scenarios.

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